ARISTARCHUS'S BOOK ON THE SIZES AND DISTANCES OF THE SUN AND OF THE MOON

WITH SOME EXPLANATIONS OF PAPPUS ALEXANDRINUS.
IN LATIN TRANSLATED AND ILLUSTRATED WITH COMMENTARY FROM FEDERICO COMMANDINO OF URBINO

P E S A R O, by Camillus Francischinus
M D L X X I I

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Introduction, translation and notes
by Antonio Mancini

Latin text in front

‘Consider ye the seed from which ye sprang:
ye were not made to live like unto brutes,
but for pursuit of virtute and knowledge.
So eager did I render my companions,
with this brief exortation, for the voyage,
that then hardly could have held them back.

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Dante, The Inferno XXVI

In memory of Emma Castelnuovo

Acknowledgments

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INTRODUCTION

I FEDERICO COMMANDINO AND THE DISCLOSURE OF THE ANTIQUE WRITTEN OF MATHEMATICS

The evolutionary impulsion imposed on the present society by the spread of computer technology reminds us of the great acceleration that the Gutenberg's movable types printing invention, in the mid-fifteenth century, gave to the knowledges diffusion, which at the end of the century, facilitated the so-called scientific revolution. In the last years of the fifteenth century hundreds of thousands of books, writin with this new technique, were circulating, and many literates, artisans, soldiers, architects and students could easily access texts unavailable in the past for their rarity, as handwritten or products with primitive print shapes.

In the sixteenth century the books took on a form more resembling to the modern one. They were no longer a simple translation of handwritten texts, but they had a new formatting: author, title, frontispiece, dedication, privilege, etc. The power of the new invention increased interest in ancient writings of mathematics, astronomy and natural philosophy. Numerous authors devoted their intelligence to the print edition of ancient manuscripts. In 1482 the Euclid's Elements came out for the first time in Venice by Erhardus Ratdolt's printing-shop[1]. Niccolò Fontana from Brescia, better known as Niccolò Tartaglia, translated "Euclid's Elements for common use and utility from Latin to Vulgar" and published them for the first time in 1543 in Venice [2]. Thomas Gechauff, better known as Thomas Venatorius, published in Basel in 1544 the complete Archimedes work in Greek[3]

In Italy Federico Commandino published many greek ancient mathematicians works into latin, reaching a clear fame.[4] Federico was born in Urbino in 1509 and the cultural climate of the city certainly had an important role in his intellectual development. His father, Giovan Battista, had been the military architect who, on behalf of Francesco Maria I della Rovere, had reinforced Urbino's walls to adapt them to resist to the growing artillery, his grandfather had been secretary of Federico da Montefeltro (†1482). This prince in the course of his life besides the art of war, had cultivated liberals arts by protecting artists and literates and had created a library that was considered one of the most illustrious. Federico Commandino studied medicine in Padua by completing his studies in Ferrara. He became the personal physician of Cardinal Ranuccio Farnese, brother-in-law of Guidobaldo della Rovere Duke of Urbino, from wich he gained protection. During his life, however, he was especially attracted to mathematics, and devoted his intellectual work to the edition of ancient texts, transcribing them from manuscripts that, being received through the long medieval ford, were sometimes in poor conditions, as in the case of De Magnitudinibus.

In 1558 Commandino published the *Archimedis Opera non nulla* containing Circuli Dimensio (cum commentariis Eutocii Ascalonitae), De Lineis Spiralibus, Quadratura Paraboles, De Conoidibus, & Sphaeroidibus, De Arenae Numero. In the same year he published *Ptolemaei Planisphaerium Iordani Planisferium Ptolemaei Planisphaerium Commentarius*, elaborating a text of 1536, printed in Basel by Johann Walder, this book included several works, among which the Ptolemy planisphere, drawn from a Latin manuscript dating from Tholosae Calendis Iunii anno domini MCXLIIII in turn version of an arabic text, and the *De planisphaerij figuratione* of Jordanus Nemorarius, which treat about the projection stereographic of the celestial vault on the equatorial plane. In 1562 he published in Rome the Liber de Analemmate of Ptolemy, a work dealing with gnomonics, that is the study of the solar positions. It is possible in fact to obtain the local time from the position of the sun, detecting the shadow projected by a reference, called gnomon, on a flat surface; the simplest gnomon is made up of a shaft fixed in the ground that projects its own shadow over the surrounding area. It is known that in the construction of solar clocks, commonly called sundials, it must be borne in mind that the shadow of the gnomon varies not only with the hours passing, but also with the latitude of place and with the seasons; these different parameters must therefore be known and correctly used by the designer. In the famous solar clock of the San Petronio Basilica in Bologna, designed by Egnazio Danti, rebuilt and expanded in 1655 by Gian Domenico Cassini, instead of a shadow, an eye of light is projected onto the floor surface, generated by a small hole in a wall perimeter of the church. Solar clocks of this size also allow to carry out with great accuracy other measurements taken from the position of the sun, with which the equinoxes, the cardinal points, the length of the tropical year etc. are determined. To the book of Ptolemy Commandino also adds a personal contribution for the realization of such instruments. The author did not have a Greek text and he used a Latin translation of a Arabic text. In 1565 he published the Archimedes' treatise on floating bodies and in the same year the *Liber de centro gravitatis solidorum.* The Commandino as translator is consolidated as an author; in fact, in the dedication to Cardinal Alessandro Farnese we read that he examines “perdifficilis et perobscura quaestio de centro gravitatis corporum”, he then extends its search to the center of gravity of solid figures. Indeed only in 1908 will it be discovered by Heiberg in Constantinople the palimpsest that contained the Method on mechanical theorems, a writing of Archimedes to Eratosthenes, in which are also determined the barycenters of some solids [5]. The definition of center of gravity exhibited at the work beginning, is drawn from the eighth book of the *Mathematicae Collectiones* by Pappus and is reported by Commandino both in Greek and in Latin version: “*Dicimus autem centrum grauitatis uniuscuiusque corporis punctum quoddam intra positum, à quo si graue appensum mente concipiatur, dum fertur quiescit; & seruat eam, quam in principio habebat positionem: neque in ipsa latione circumuertitur*”. [6]

In 1566 he published in Bologna *Apolloni Pergaei Conicorum libri quattuor* e *Sereni Antinsensis philosophi libri duo.*

Commandino, who returned to Urbino after the unexpected death in 1565 of Cardinal Ranuccio Farnese, met in 1570 the Joannis Dee from London, a mathematician and lover of esotericism, who had collaborated on the publication of the Elements of Euclid in English. Dee had the Latin translation of Euclid's text on the division of flat figures based on a manuscript in Arabic, this work was published in Pesaro with the two authors' names. The work, written in Latin and dedicated to Francesco Maria II della Rovere, was also translated into Italian by Fulvio Viani: "*or persuadendomi adunque che ella se è piaciuta a V E. ne l'habito latino, non habbia a dispiacerle in questo nostro vulgare*" [7].

In 1572 he published *Euclidis Elementorum libri XV*, his most famous work, on request (rogatu iussuque) by Francesco Maria II della Rovere, Duke of Urbino, a Latin translation of Greek manuscripts.

In the same year he published *Aristarchi de magnitudinibus et distantiis solis et lunae liber.*

In 1575 the Elements were also published in Italian, from the dedication to the Duke of Urbino, signed by the son-in-law Valerio Spacciuoli, we learn that Commandino had just the time, before dying, to see the printed output. This Italian version, edited by his pupils under his supervision, had been requested by many to allow access to this work also to those who are not familiar with the Latin language. Moreover, the author had always said that the only aim of his life, and also why he had abandoned the practice of medicine, had been to come to the aid of those who wanted to devote himself to mathematical studies

In 1575 it was also published posthumously *Heronis Alexandrini Spiritalium liber*, a short brochure that deals with various pneumatic devices such as siphons. In 1588, Guidobaldo dal Monte, his favorite student, published the *Matematicae Collectiones* of Pappus with the commentaries of the Commandino. Editions of his works have continued in the following centuries edited in various countries by different authors.

II ARISTARCHUS OF SAMOS

The book on the magnitudes and distances of the sun and the moon is the only work of Aristarchus that has come to us. Vitruvius, in the first book De Architectura, counts him among the seven great mathematicians of the past together with Philolaus and Archytas of Tarentum, Apollonius of Perga, Eratosthenes of Cyrene, Archimedes and Scopas of Syracuse. We know that Aristarchus, mathematician and astronomer, lived before Archimedes, who died in 212 BC, because the great Syracusan dedicates to him some passages of the famous script Ψαμμίτης (Psammites or, in the Latin version, De arene numero or Arenarius[8]) in which he proposes to calculate the number of sand grains which can be contained in the whole universe. The work of Aristarchus has survived through numerous manuscripts in Greek, the oldest and most famous of which is contained in cod. Vaticanus Græcus 204 of the tenth century.[9], but Commandino does not refer to which sources he drew for his translation.

Remarkable is the impact that the work produces on the reader; the hypotheses necessary for the subsequent demonstrations are in fact proposed in a concise, immediate and axiomatic form, surprising, if one thinks, they were formulated in the 3rd century BC. Based on these assumptions, Aristarchus builds a credible theory adapted to evaluate not easily quantifiable cosmic dimensions, rather commonly perceived as mysterious, and so he opens the way to further speculations. In his short treatise he does not mention any personal observation of the sky, indeed he much less care to provide an evidentiary basis for his assumptions, indeed his intent is to prove, with mathematical reasoning, some propositions from the initial axiomatic statements. Essentially the correctness of his theory relates to the assumptions, and any quantitative changes to the same hypotheses involve a quantitative change of the demonstrated deductions. This fact deserves some considerations.

The sizes and distances of the sun and the moon calculated by Aristarchus are not close to those we find today, the average distance of the sun from the earth, for example, is today valued at approximately 390 times the average distance of the moon from the earth and not 18-20 times, as instead Aristarchus had calculated and this happens not for lack of mathematical proof but because the assumption of the fourth hypothesis of De Magnitudinibus, i.e. the estimation of the moon elongation at quadrature[10] (the instant of the first or last quarter), it turns out, to today's technically advanced observations, of 89 ° 51 'and not 87°; this difference, less than 3 °, involves a considerable underestimation of the earth to sun distance[11]. Howewer we must consider the fact that determining the exact moment in which the moon is at the first or last quarter, to measure its elongation, is not easy without high precision instruments.

But there is another objection that can be raised to Aristarchus: having incorrect in the evaluation of the lunar diameter equal, according to him, to one-fifteenth of a zodiac sign that is 2 °. This measure could easily be determined with greater accuracy even with simple technical means (just look at how long the lunar disk passes through a taut wire), so much the same Pappus, in the commentary to the initial hypotheses of Aristarchus, reports that Hipparchus evaluates the width of the moon in about 0 ° 27 'and Ptolemy in about 0 ° 31'. It is evident that Aristarchus did not give significant weight to the accuracy of the astronomical measurements indicated by him, since he has the intention to expose a calculation method that could be later used with greater accuracy. Today we would say that he was a theorist and not an experimental one.

Thomas Heath[12] points out that in the work of Aristarchus important relationships between angles and sides of a triangle are already being indicated; in proposition VII, for example, he determines the approximate value - between 1/20 and 1/8 - of the tan of 3°.The well known trigonometric functions - sin α, cos α, tan α etc - they will be exhibited in later times, but it is during the Hellenism that spherical and flat trigonometry are born with the calculation of the circumference strings as a function of the angles at the center that subtended them.[13] Aristarchus is mainly known because he was one of the first supporters of the heliocentric theory opposed to the geocentric theory instead supported by most astronomers of his time and also of the later times; yet in this one work which he has received, he makes no heliocentric hypothesis. We know about this position from other authors and the first to give us this information was Archimedes who writes about it in Ψαμμίτης: "*He (Aristarchus) in fact supposes that both the fixed stars and the sun itself are motionless, while the earth is carried along a circumference around the sun, which is located at the center of the circular path, and that the same sphere of fixed stars, situated around the same center together with the sun, is so great that the circumference along which he supposes that the earth to moves, is in relation to the distance of the fixed stars as the center of the sphere relative to the surface. But now it is clear that this cannot be, because not having the center of the sphere any dimension cannot be conceived any relationship with the surface of the sphere. We must instead assume that Aristarchus wanted to say this: since we believe that the earth is at the center of the universe, the ratio that the earth will have to what we call the universe is equal to the ratio that the sphere containing the circle, in which he supposes that the earth turns, has to the sphere of the fixed stars.*"[14]

In his short treatise on the distances of the sun and moon there is therefore no trace of this theory that actually appears is not influential for the methodology used by the author to evaluate the distances of the sun and moon from the earth. Thomas L. Heath in his classic book on Aristarchus believes that two reasons may explain the absence of the heliocentric hypothesis in this work: that Aristarchus's acceptance of this hypothesis is subsequent to the writing of the book or that this hypothesis is irrelevant. This second reason seems more consistent because the heliocentric hypothesis is prior to Aristarchus and it is difficult to think that he had not known it or had not yet accepted it at the time of writing of the De Magnitudinibus. Copernicus resumed the heliocentric theory many centuries after Aristarchus, exposing it in a convincing and organic way in the *De Revolutionibus Orbium Coelestium Libri VI*. In this cosmic system the earth behaves like a planet or like a wandering celestial body. The heliocentric opinion of Aristarchus, in the past considered a precursor of Copernicus, cannot but remind us of the vexed question if it was the sun to orbit around the earth or the opposite. This has never been the problem such even if it has been described and emphasized as such. Copernicus waited many years before deciding to publish his books. This delay was probably not due to the lack of conviction for his theory rather for the fact that he knew that there would be a strong reaction from conservative circles. In fact Copernicus, in his dedication to Pope Paul III, says: "However, in order that the educated and uneducated people alike may see that I do not run away from the judgement of anybody at all, I have preferred dedicating my carefull studies to Your Holiness rather than to anyone else. For even in this very remote corner of the earth where I live you are considered the highest authority by virtue of the loftiness of your office and your love for all literature and mathematics too. Hence by your prestige and judgement you can easily suppress calumnious attacks although, as the proverb hasit, there is no remedy for a backbiter." When the first edition took place in 1543, Copernicus was dying. A preface was added to his work to reduce the blow he might have had on the dominant opinion; in fact the Copernican script was presented as a pure mathematical hypothesis, thus depriving the so-called real physical meaning. This preface, later authoritatively revealed apocryphal by Kepler, was added by Andreas Hosemann, better known as Osiander, the reformed theologian who was in charge of editing the edition by the aid of Georg Joachim von Lauchen, professor of mathematics, also better known with the Latinized name of Reticus; he was the one who had long urged Copernicus to publish his work. But does it make sense to say that the Copernican theory is only a mathematical hypothesis devoid of real physical content? If we believe that the physical content consists in the descriptive and predictive capacity of events controllable by experience (the classic φαινόμενα ςώζειν: theories must "safeguard the phenomena", that is, be adherent to observational data, in other words be compatible with experimental observations)[15] then the Copernican hypothesis is not without it, since the data contained in the De Revolutionibus were used in 1545 for the publication of the Tabulae Prutenicæ; these astronomical tables were used by the commission for the

reform of the Julian calendar commissioned by Pope Gregory XIII in 1582.
In short, the Copernican theory worked as a practical tool.

The system described by Ptolemy in the Almagest was, among the precopernican cosmologies, the most accredited; it described the movements of celestial bodies taking our earth as a fixed reference for the description of their motion. The fixed stars (stellae inerrantes i.e. those which maintain their relative positions), in this representation, seem to move as if they were pinned on a celestial sphere rotating on its axis and drag this into a daily tour; the sun motion can be described as occurring along a particular circular orbit called the ecliptic and the motion of the planets is described as the result of particular traces called deferents and epicycles.

However, this theory, in the description of the Almagest, proved insufficient to describe planetary motion because it was not consistent with some astronomical observations. Copernicus, in this regard, in the dedication to Paul III states: "*So I do not want to hide from Your Holiness thath I was impelled to consider a different system of deducing the motions of the universe's spheres for not other reason that I realize that Mathematicians do not agree among themselves in their investigations*". The intuition of Copernicus was therefore to change the usual system of terrestrial reference, that any observer, in solidarity with the earth, assumed automatically and perhaps unconsciously as the only possible, and consider the sun as a new fixed reference, describing in a different way the motion of the planets and the earth compared to the same. It follows that the earth becomes a planet, which is quite acceptable in a scientific reasoning, but evidently difficult for the common sense of the time, almost like the term planet, rather than describing a movement with respect to a reference that is assumed to be immobile, was offensive towards the earth and its inhabitants. In essence, the earth is a planet if we take the sun as a reference for the motion of the celestial bodies, while it is not if we consider it fixed or it is itself the reference of the astral motion. The algorithms that describe the motion of the planets with optimal approximation to the solar reference are elementary functions, while the algorithms that describe the motion of the planets by taking the earth as a reference are more complex. The theory of Copernicus, which modified the immobile reference and the consequent new description of planetary motions, also contained erroneous assertions, such as the circular pattern of planetary orbits and their uniform motion with respect to the heliocentric reference, but certainly this does not diminish its importance. It was Kepler, after a few decades, to make some changes to the Copernican theory by demonstrating that astronomical measurements on the motion of the planets, in a heliostat reference system, give evidence of non-circular orbits but elliptical, with the sun situated in one of the geometric fires of ellipses, and for the uneven motion of the planets around the sun,[16] the Copernican theory, rightly called revolutionary because it is the bearer of important developments, has in fact constituted a great advancement in cosmic knowledge overcoming the millennial stalemate. Aristarchus seems to write as if he already intuited the relativism of motion; his propositions are valid both in a heliocentric reference system as in a geocentric one, he is therefore careful not to introduce in his reasoning distortive elements, extraneous to the hypotheses and irrelevant to the propositions to be demonstrated. Newton exposing the regulæ philosophandi in Philosophiæ Naturalis Principia Mathematica – De Mundi Systemate – Book III - will write many centuries later: Rule I: *We are to admit no more causes of natural things than such as are both true and sufficient to explain their appearances. To this purpose the philosophers say that Nature does nothing in vain, and is in vain make use more things than will serve; for Nature is pleased with simplicity, and affects not the pomp of superfluous causes*.[17]

It is striking in Aristarchus the aptitude for calculation with large numbers using a numbering system certainly less convenient than the current. In proposition XVIII, for example, he calculates that the volume of the earth with respect to the lunar volume is between ratios $\frac{1259712}{79507}$ e $\frac{216000}{6859}$. These numbers appear to us easily understandable because in translations for printed editions are transcribed with the Indo-Arabic numerals, using this notation in our decimal system such ratios, approximate to the third digit, we can simply write 15.844 and 31.491. The ancient notation is obviously used in the Greek manuscripts, so these same numbers are written like this:[18] [M ^{ρκε}, θψιβ]; this reads: ρ=100, κ= 20, ε=5 while M indicates that each number is multiplied by 10,000 ie 1.000.000, 200.000, 50.000 seguito da θ=9, ψ=700, ι=10, β=2 ossia in totale 1.000.000 + 200.000 + 50.000 + 9(000) + 700 +10 + 2 = 1.259.712 ; analogically [M^{ζ},θφζ] M^{ζ}=70.000, θ=9, φ=500, ζ=7 ie 70.000+9(000)+500+7=79.507; the comprehensibility of this notation is obviously less easy.

A. M.

ARISTARCHI
DE MAGNITVDINIBVS, ET DISTANTIIS SOLIS, ET LVNAE, LIBER

CVM PAPPI ALEXANDRINI explicationibus quibusdam.

A FEDERICO COMMANDINO Vrbinate in latinum conuersus, ac commentarijs illustratus.

Cum Priuilegio Pont. Max. In annos X.

PISAVRI, Apud Camillum Francischinum.
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Latin text in front

ILL.^{MO} AC NOBILISS.^{MO}
ALDERANO CIBO
MALASPINÆ
CARRARIÆ MARCHIONI.

POST Euclidis elementa typis excuſa, in quorum quidem editione, rogatu iuſſuq; FRANCISCI MARIAE Principis Illuſtriſſimi ſuſcepta, cui ego & otium, & ſtudia omnia deuoui mea, induſtriæ atque laboris plurimum impendi, non inepte me facturum exiſtimaui, Clariſsime ALDERANE, ſi alium mox libellum planè aureum, ac vetuſtiſsimum, à præſtantiſsimoq; philoſopho Ariſtarcho de Solis & Lunæ magnitudine, ac diſtantia conſcriptum, diuulgandum proponerem. qui mihi tum ob argumenti præſtantiam & dignitatem, tum ob ſingularem auctoris ſolertiam, ac diuinam propè ingenij fœlicitatem viſus eſt non indignus, qui à tot annorum ſitu, & ſqualore reuiuiſcens in doctiſsimorum hominum, &

✠ 2 *præſertim*

TO THE MOST ILLUSTRIOUS AND NOBILE ALDERAN CIBO MALASPINA MARQUIS OF CARRARA

After the printing of the Euclides Elements, to whose edition i really devoted much effort and carefulness, cared for desire and charge of FRANCESCO MARIA most illustrious prince, sacrificing my free time and my occupation , I trought did not go wrong, clear ALDERANO, with the intention to disclose another small, very precious and ancient book, written by the famous philosopher Aristarchus about magnitudes and distances of the sun and of the moon, which, for the importance and dignity of the subject, and for the singular ability of the author and for the almost divine excellence of his wisdom, I thougth deserved to come to the hands of the men of science , specially mathematicians, reviving from the decay and præſertim mathematicorum manus perueniret. Verumenimuero male cum ipſo actum eſt. vel enim temporum, vel librariorum, vel amborum potius iniuria, & inſcitia tam miſere labefactatus, turpiterq́ deformatus fuit, (quod ſanè malum in omnes paulo vetuſtiores libros magno doctorum incommodo & iactura latius ſerpſit) vt mihi nunc, qui eius ulcera ſanaui, maculasq́ abſterſi, & meis in ipſum conſcriptis commentarijs exornaui, ſtudij fortaſſe & vigilantiæ non minus ponendum fuerit in hoc opere, quàm ipſe ab initio poſuerit Ariſtarchus. Hunc igitur mea induſtria in priſtinum nitorem reſtitutum, & perpolitum, latinitateq́ donatum, unà cum Pappi Alexandrini explicationibus quibuſdam, ſub tui Illuſtriſsimi nominis tutela, & patrocinio in lucem prodire volui, tum vt mei perpetui erga te amoris, atque obſeruantiæ ſpecimen hoc eſſet, cum nulla alia ratione, quanti te faciam, quantumq́ in præſtantiſsima natura, eximioq́, ac ſingulari ingenio confidam tuo, declarare nunc liceat; tum vt tu, qui, ſummo loco natus, in magno generis

ſplendore,

carelessness of so many years. This book was however treated really badly. In fact this book, for the damage of the time or of the copyists, or better of both, as well as for incompetence, has been so miserably damaged and obliviously deformed (this evil indeed spread to a very large extent in all the books that are not very old with great inconvenience and harm to the learned men) thath perhaps, now that I have healed her sores and cleaned her stains and adorned of my commentaries attached to it, I have put, on my side, in this work unlessened study and attention than Aristarchus himself has placed they at the beginning.

So I wanted this book, returned to the original splendor, purified and donated by my work to the Latin language toghether with some explations of Pappus of Alexandria, to be revealed under the protection and patronage of your illustrious name, both for this to be an example of my perpetual affection and respect towards you, so that I may now show, without any other interest, how much you I exteem and how much I trust in your excellent nature and in your distinguished and singular ingenuity, both so that you, born in very high position, in great splendor of family, surrounded by atavistic glory, wealth, dignity, ſplendore, & maiorum gloria, opibus, dignitate, gratia circumfluens, & virtutum omnium, atque artium optimarum miro incenſus ardore, in quibus & tua ſponte, & ſtudio, ſingulariq; conſtantia adeo proceßiſti, vt nihil non amplum, non ſummum, non glorioſum de te ſperandum ſit, mathematicas diſciplinas, quarum te incredibili deſiderio flagrare noui, hac ratione habeas quàm commendatiſſimas, & magno præſidio tuearis. Inſignem autem, & egregium mathematicum fuiſſe Ariſtarchum, non ſcripta eius tantum aperte teſtantur, in quibus tametſi alia methodo, aliiſq; poſitionibus nixus, atque Hipparchus, & Ptolemæus eadem in re vſi fuerint, ſcientiam ſempiternorum corporum, nobiliſſimam illam quidem, & vehementer expetendam, longiſſime tamen à communi hominum ſenſu poſitam, egregie, vt temporibus illis, aſſecutus fuit, & luculenter explicauit; ſed ipſius etiam Archimedis in libro de Arenæ numero teſtimonium ampliſſimum, & locupletiſſimum. neque enim vir ille Diuinus Ariſtarchum tot in locis laudaſſet, niſi hominis *grace, inflamed with admirable ardor for all the viertues and sciences in wich, by your will and commitment and by singular constancy, you are so advanced that we can not hope from you for anything that is not great, supreme, glorious, have for this reason in high regard and protect with great force the mathematical disciplines for wich I know that you burn with incredible passion. That Aristarchus was then a eminent and distinguished mathematician not only is attest by his writings, in which he, throug by another method and based on different hypotheses, understood and explained, very well and widely for those times, the science of eternal bodies, matter in which both Hipparchus and Ptolemy later will be experts, althrough it is very noble and to be sought after strongly yet so far from the common sense of men, but also witness to him in a very relevant and authoritative way the book of Archimedes De Arenae numero.[19] In fact, that divine man would not have praised Aristarchus in many passages if is doctrine hat not* *nis doctrina sibi spectata, probataq; fuisset. Adde quod Sami ortum testificatur; quæ insula, vrbsq; olim Pythagoram tulerat omnium liberalium artium uel repertorem, uel certe doctorem præstantissimum, ac mathematicis ita deditum, ut, cum in Geometria noui quiddam inuenisset, musis bouem immolasse dicatur. Hunc inprimis ab Aristarcho magistrum sibi lectum credi facile potest: etenim viri laudis amantes ciuium suorum, quorum nomen celebre uident, uestigijs ad gloriam alacrius incedunt. Accipe igitur hoc à me munusculum, & perfruere, Commandini tui non immemor, qui unice colit & obseruat. Vale.*

Federicus Commandinus.

been known and approwed by himself. Moreover, he testifies that he was born in Samos; which island and city had had in the past Pythagoras, inventor of all liberal arts and also a very capable teacher, he was so devoted to mathematics that it was said that, when he discovered somethings new in Geometry, he would sacrifice an ox to the muses. It is easy to believe that he was chosen especially by Aristarchus as his teacher: in fact men who lowe the praise of their fellow citizen, whose name they see celebrated, are more eagerly moving towards glory. So welcome my little gift and enjoy remembering your Commandino, who honors and respect you in an extraordinary way. You're fine.

Federicus Commandinus

ARISTARCHI LIBER DE MAGNITVDINIBVS, ET DISTANTIIS SOLIS, ET LVNAE,

VNA CVM PAPPI ALEXANDRINI.

Et Federici Commandini Commentarijs.

POSITIONES.

LVNAM à Sole lumen accipere. 1

Terram puncti, ac centri habere rationem ad sphæram lunę. 2

Cum luna dimidiata nobis apparet, uergere in nostrũ visum circulum maximum, qui lunæ opacũ, & splendidum determinat. 3

Cum luna dimidiata nobis apparet, tunc eam à sole distare minus quadrante, quadrãtis parte trigesima. 4

Aristarchus's book on the sizes and distances of the sun and of the moon.

With commentary from Pappus Alexandrinus and from Federico Commandino

HYPOTHESES.

1. *The moon receives light from the sun*
2. *The earth is in the relation of a point and centre to the sphere of the moon*[20].
3. *When the moon appears to us halved, the great circle which divides the dark and the bright portions of the moon is in the direction of our eye*[21].
4. *When the moon appears to us halved, its distance from the sun is then less than a quadrant by one – thirtieth of a quadrant.*

5 *vmbræ latitudinem esse duarum lunarũ.*
6 *Lunam subtendere quintam decimam partem signi.*

Itaque colligitur, Distantiam solis à terra, maiorem quidem esse, quàm duodeuigintuplam distãtiæ lunę; minorem vero quàm vigintuplam, ex positione, quæ est circa dimidiatam lunam: et eandẽ proportionem habere solis diametrum ad diametrum lunę. Solis autem diametrum ad diametrum terrę maiorem quidem proportionem habere, quã 19 ad 3; minorem vero quàm 43 ad 6, ex ratione distantiarum, & positione circa vmbram, & ex eo quòd luna quintam decimam signi partem subtendit.

Pappus in sexto libro collectionum Mathematicarum.

Aristarchus, inquit, in libro de magnitudinibus, et distãtijs lis & lunę sex ponit, nẽpe hęc, Primũ, lunam à sole lumen accipere. Secundum, terram puncti ac centri habere rationem ad spheram lunę. Tertium, cum luna dimidiata nobis apparet, vergere in nostrum visum circulum maximum, qui lunę opacum, & splendidum determinat. Quartum, cum luna dimidiata nobis apparet, tunc ipsam à sole distare minus quadrante, quadrantis parte trigesima pro eo, quod est distare partes octaginta septem, hę enim minores sunt, quàm nonaginta partes quadrãtis, partibus tribus, quae sunt trigesima pars nonaginta. Quintum, vmbrae latitudinem esse duarum lunarum. Sextum, lunam subtendere quintam decimam partem signi.

Harum

5.

5. *The breadth of the shadow is that of two moons.*
6. *The moon subtends one fifteenth part of a sign of the zodiac.*

From this it is deduced that the distance of the sun is greater than eighteen times, but the distance of the moon less than twenty times, this follows from the hypothesis about the halved moon, and thath the diameter of the sun has the same ratio to the diameter of the moon. But still we deduce that the diameter of the sun has a ratio with respect to the diameter of the earth certainly greater than 19 to 3 but less than 43 to 6, because of the relationship of distances, for the hypothesis on the shadow and for the fact that the moon subtends the fifteenth part of a zodiacal sign.

Pappus in the sixth book of Matematicae Collectiones

Aristarchus, in his book on sizes and distances of the moon and of the sun, lays down these six hypothesis:

1. *That the moon receives light from the sun*
2. *That the earth is in the relation of a point and centre to the sphere of the moon*
3. *That, when the moon appears to us halved, the great circle which divides the dark and the bright portions of the moon is in the direction of our eye.*
4. *That, when the moon appears to us halved, its distance from the sun is then less than a quadrant minus one thirtieth of a quadrant, i.e. eighty-seven times, these are actually smaller for three parts, compared to ninety parts of a quadrant, which are the thirtieth part of ninety when the moon appears to us halved, its distances from the sun is then less than a quadrant minus one-thirthiet of a quadrant, i.e. eighty-seven times, these are actually smaller for three parts, compared to ninety parts of the a quadrant, which are the thirtieth part of ninety*
5. *That the breadth of the shadow is of two moons*[22]
6. *That the moon subtends one fifteenth part of the zodiac.*

Harum autem poſitionum, prima quidem, tertia & quarta ferè cum Hipparchi & Ptolemęi poſitionibus conſentiunt; luna enim à ſole ſemper illuminatur, pręterquàm in ecclipſi: quo tempore lucis expers fit, incidens in vmbram, quam ſol oppoſitus à terra iacit, conicam formam habētem, & circulus determinans lacteum, quod eſt ex illuminatione ſolis, & cineritium, qui proprius lunę color eſt, haud differens à maximo circulo in dimidiatis ad ſolem conſtitutionibus, quàm proxime ad quadrantem in zodiaco conſpectum, ad viſum noſtrum vergit. hoc enim circuli planum, ſi producatur etiam per viſum noſtrum tranſibit, quamcumque poſitionem habeat luna primę, vel ſecundę dimidiatę apparitionis. reliquas autem poſitiones diſcrepantes comperierunt dicti mathematici, propterea quòd neque terra puncti, ac centri rationem habeat ad lunę ſphęram, ſecundum ipſos, ſed ad ſphęram inerrantium ſtellarum. neque vmbrę latitudo ſit duarum diametrorum lunae: neque ipſius lunę diameter ſubtendat circumferentiam maximi circuli, ſecundum mediam eius diſtantiam, quintam decimam partem ſigni, videlicet partes duas. Hipparcho enim diameter lunę circulum hunc ſexcenties & quinquagies metitur: & circulum vmbrę metitur bis & ſemis ſecundum mediam diſtantiam in coniunctionibus. At Ptolomęo diameter ipſius lunę ſecundum maximam quidem diſtantiam ſubtendit circumferentiam 0. 31. 20. ſecundum minimam vero 0. 35. 20. Et diameter circuli vmbrę ſecundum maximam lunae diſtantiam 0. 45. 38. ſecundum minimam. 0. 46. Vnde ipſi differentes rationes tum diſtantiarum tum magnitudinum ſolis & lunę collegerunt. Ariſtarchus enim dictas poſitiones ſecutus ad verbum ita ſcribit.

Itaque colligitur diſtantiam ſolis à terra maio- ,,
rem quidem eſſe, quàm duodeuigintuplam diſtan- ,,
tiæ lunæ; minorem vero, quàm vigintuplam: & ,,

The first, third, and fourth of these hypothesis agree pretty good with the hypothesis of Hipparcus and Ptolemy; indeed the moon is illuminated by the sun at all times except during an eclipse, when it becomes devoid of light sinking into the conical shadow that the sun in opposition throws from the earth, and the circle, dividing the milk-white portion, caused by the sun, from ashy portion, wich is the color of the moon, is in the direction of our eye, itself indistinguishable from a great circle observed when moon, in relation to the sun position, appear halved very nearly a quadrant in the zodiac. This plane of the circle, if produced, will in fact pass through our eye in whatever position the moon is when for the first or second time it appears halved. The aforementioned mathematicians, as regard the remaining hypothesis, have taken a different position, since, for according to them, the earth has the relation of a point and centre to the sphere of the fixed stars but neither to the sphere in wich the moon moves, nor the shadows breadth is that of two moons diameters[23], nor the moon's diameter subtend one fiftheenth part of a sign of the zodiacal circle, at its average distance, that is 2 parts[24]. According to Hipparcus, the moon's diameter is contained 650 times into this circle and 2 ½ times into shadow's circle, at its mean distance in the conjunctions[25]. According to Ptolemy the moon's diameter subtends a circumference of 0.31.20, when the moon is at its greatest distance, and 0.35.20 when at its least distance. The diameter of the shadow's circle[26] 0.45.38 when the moon is at its greatest distance, 0.46 when the moon is at its least distance. Hence themselves calculated different ratios of both the distances and of the sizes of the sun and the moon. Therefore Aristarchus, stating the aforeside hipotheses, writes word for word:

Now we can prove that the distance of the sun from the earth is greater than 18 times, but less than 20 times, the distance of the moon, and the diameter of the sun has the same ratio to the diameter of the moon: this follows from the hypothesis about the halved moon.

„ eandem proportionem habere ſolis diametrum ad
„ diametrum lunæ. quod habetur ex poſitione, quæ
„ eſt circa dimidiatam lunam. ſolis autem diametrũ
„ ad diametrũ terrę in maiori proportione eſſe, quàm
„ 19 ad 3, & in minori, quàm 43 ad 6, ex ratione di
„ ſtantiarum, & poſitione circa vmbram, & ex eo
„ quòd luna quintamdecimam ſigni partem ſub-
„ tendit.

Colligitur inquit, vt deinceps, velut qui hęc paulo poſt demonſtraturus ſit, lemmata ad demonſtrationes vtilia pręmittens. Ex quibus omnibus concludit, ſolem ad terram maiorẽ quidem proportionem habere, quàm 6859 ad 27; minorẽ vero, quàm 79507 ad 216. Terrae diametrum ad diametrum lunę in maiori proportione eſſe, quàm 108 ad 43; & minori, quàm 60 ad 19. Terram vero ad lunam in maiori eſſe proportione, quàm 1259712 ad 79507; & minori, quàm 216000 ad 6859. At Ptolemęus in quinto libro magnae conſtructionis demonſtrauit quarum partium ſemidiameter terrae eſt unius, earum lunae maximam diſtantiam in coniunctionibus eſſe 64. 10. & ſolis 1210. ſemidiametrũ lunae 0. 17. 33. & ſemidiametrum ſolis 5. 30. ergo quarum partium diameter lunae eſt unius, earum diameter quidem terrae eſt 3 $\frac{2}{5}$: ſolis autem 18 $\frac{4}{5}$. terrae igitur diameter tripla eſt diametri lunae, & adhuc duabus quintis maior. ſolis diameter diametri quidem lunae duodeuigintupla eſt, & adhuc maior quattuor quintis: diameter autem terrae quintupla, & adhuc dimidio maior. Ex quibus & ſolidorũ corporum proportiones manifeſtę ſunt. Quoniam enim cubus vnius eſt 1, cubus aũt 3 $\frac{2}{5}$ eſt earũdẽ 39 $\frac{1}{4}$ ꝓximè; et cubus 18 $\frac{4}{5}$ ſimiliter 6644 $\frac{1}{2}$ proximè: quarum partiũ lunae ſolida magnitudo eſt unius, earum magnitudo terrae erit 39 $\frac{1}{4}$; & ſolis 6644 $\frac{1}{2}$. Quare magnitudo ſolis centies & ſeptuagies proximè terrae magnitudinem continet.

&

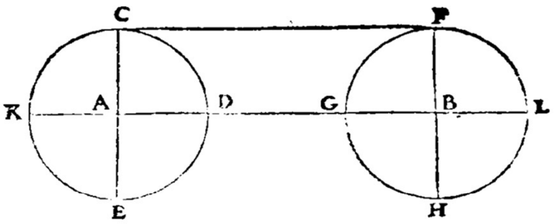
Again the diameter of the sun is to diameter to the earth in a greater ratio than that wich 19 has to 3, but in a less ratio than that which 43 has to 6; this follows from distances ratio, from the hypothesis about the halved moon and from the hypothesis that the moon subtends one fifteenth part of a zodiacal sign. *He sais: "it can be deduced", that does imply that he will to prove these things, after giving the preliminary lemmas usefull for the proofs. From all these tings he concludes that the sun has to the earth a ratio greater than that which 6859 has to 27, but a ratio greater than that wich 79507 has to 216; that the diameter of the earth is to the diameter of the moon a greater ratio than that which 108 has to 43, but in a less than that wich 60 has to 19; that the earth is to the moon in a greater ratio than that wich 1259712 has to 79507, but in a less than that wich 216000 has to 6859.*

But Ptolemy in the fifth book of Magnae Constructiones proved that, hif the half-diameter of the eart is taken as the unit, the greatest distance of the moon at the conjunctionis is 64.10, the greatest distance of the sun 1210, the half-diameter of the moon 0.17.33[27] *and the half-diameter of the sun 5.30*[28]*. In consequence, if the half-diameter of the moon is taken as the unit, the earth's diameter is 3+ 2/5 times, the sun's diameter 18+4/5 times. The earth's diameter is is 3+2/5 times the moon's diameter. The sun diameter is 18+4/5 times the moon's diameter and 5+1/2 times the diameter of the earth*[29]*. From what has been said the ratios between the solids figures are manifest. Since the cube on 1 is 1, the cube on 3+2/5 is very nearly 39+1/4 and the cube on 18+4/5 very nearly 6644+1/2: is the solid size of the moon is taken as a unit, that of the earth will be 39+1/4 and that of the sun 6644+1/2 of such units. Therefore the solid size of the sun is very nearly 170 times greater than that of the earth. This is what can be said up to now comparing the aforementioned magnitudes and distances.*

& hęc hactenus dicta sint, comparationis causa dictarum magnitudinum, & distantiarum.

PROPOSITIO. I.

Duas sphæras, æquales quidem idem cylindrus comprehendit, inæquales vero idem conus, verticem habens ad minorem sphæram: & per centrum ipsarum ducta recta linea perpendicularis est ad utrumque circulorum, in quibus cylindri, vel coni superficies sphęras contingit.

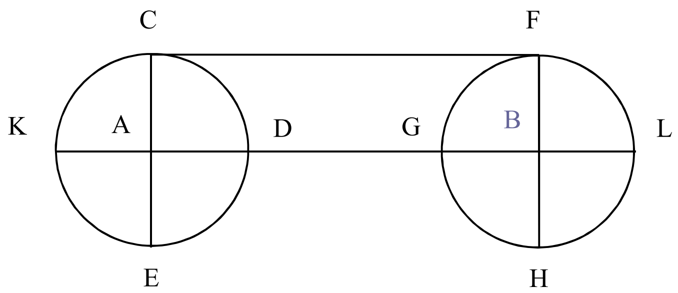


Sint æquales sphæræ, quarum centra A B: iunctaque A B producatur: & per ipsam AB producatur planum, quod faciet sectiones in sphęris maximos circulos. Itaque faciat circulos CDE FGH: atque à punctis A B ipsi AB lineæ ad rectos angulos ducãtur CAE FBH: & CF iungatur. Quoniã igitur CA FB & æquales sunt, & parallelæ, erunt & CF AB æquales, & parallelæ; eritque CFAB parallelogrammum: & anguli qui ad CF recti. ergo recta A B C

linea

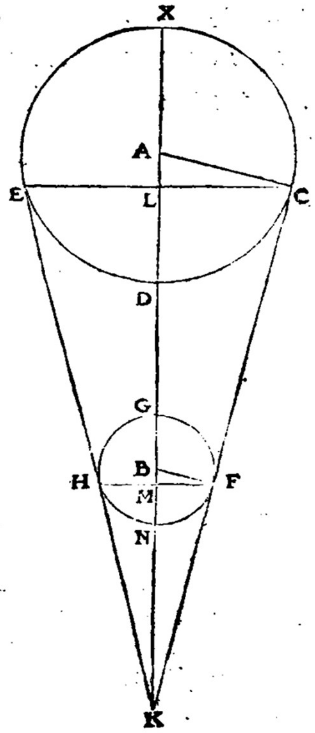
PROPOSIZION I.

One and the same cylinder comprehend two equal spheres, one and the same cone two unequal spheres wich has his vertex in the direction of the lesser sphere; and the straight-line, drawn through the centres of the spheres, is perpendicular to each of the circles in wich the surface of the cylinder, or of the cone, touches the spheres.



Let there be equal spheres and let the points A B be their centres. Let A B be joined and produced: let a plane be carried through A B this plane will cut the spheres in great circles[A.]. Let these great circles be CDE and FGH; let be drawn from A B straight-lines CAE & FBH at right angles to A B[30]; and let C F be joined. Thence, since CA & FB are equal and parallel, consequently will CF & AB be equal and parallel; CFAB will be a parallelogram: the angles at C & F will be right[B]. So the straigt-line CF touches the circles CDE &

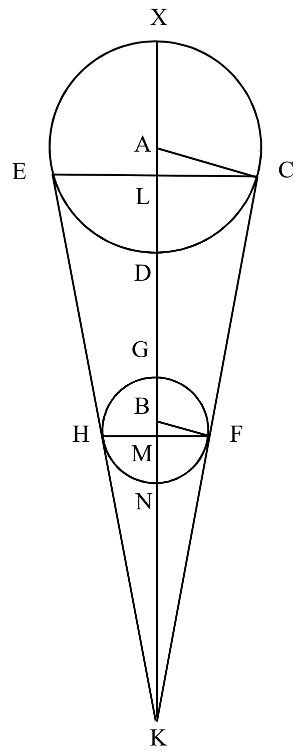
linea CF circulos CDE, FGH continget. ſi autem AB manente parallelogrammum AF, & KCD GFL ſemicirculi conuertantur, quouſque rurſus reſtituā tur in eūdem locum, à quo moueri cœperunt: ſemicirculi quidem KCD, GFL ferentur in ſphæris, parallelogrammum vero AF cylindrum efficiet, cuius
D baſes erūt circuli circa diametros CE FH, recti exiſtētes ad ipſam AB: propterea quòd in omni conuerſione CE FH ad ipſam AB rectæ permanent. Et perſpicuum eſt ſuperficiem ipſius contiugere ſphęras, quoniā CF in omni conuerſione ſemicirculos KCD GFL contingit.



Sint rurſus ſphæræ inæquales, quarū centra A B, & ſit maior, cuius centrum A. Dico dictas ſphęras eundem conum comprehendere, qui verticem habeat ad minorem, ſphæram. Iungatur AB, & per ipſam producatur planum, quod faciet
E ſectiones ī ſphęris circulos. faciat circulos CDE FGH. circulus igitur CDE maior eſt circulo FGH. ergo & quæ ex centro circuli CDE maior erit ea, quæ ex centro
F circuli FGH. fieri igitur poteſt, vt ſumatur aliqnod pun

punctum

FGH[C]. If now, AB remaining fixed, the parallelogram AF and the semicircles KCD & GFL be carried round and again restored to the position from which they started, the semicircles KCD & GFL will move in coincidence with the spheres, the parallelogram AF will generate a cylinder [D], the bases of which be the circles about the diameters CE & FH, at right angles to AB, because throughout the whole rotation CE & FH remain at right angles to AB and it is manifest that the cylinder's surface touches the spheres, since CF throughout the whole rotation touches the semicircles KCD & GFL. Again, let the spheres be unequal, and let A & B be their centres, let A the centre of greater sphere. I say that the aforeside spheres are comprehended by one and the same cone which has its vertex in the direction of the lesser sphere. Let A & B be joined, and let a plane be carried through AB, this plane will cut the spheres in circles[E]. Let the circles be CDE & FGH; therefore the circle CDE will be greather than the circle FGH. So that radius of the circle CDE is also greater than the radius of the circle FGH. Now it is possible to take a point, as



punctum, velut K, ita vt quam proportionem habet quę ex centro circuli CDE ad eam, quæ ex centro circuli FGH, eandem habeat AK ad KB. sumatur, & sit K: ducaturque KF tangens circulum FGH: & FB iungatur. Deinde per A ipsi BF parallela ducatur AC, & iũgatur CF. Quoniam igitur est, vt AK ad KB, ita AD ad BN; atque est AD quidem æqualis ipsi AC; BN vero ipsi BF: erit vt AK ad KB, ita AC ad BF: estque AC parallela ipsi BF. recta igitur linea est CFK. sed angulus KFB rectus est. ergo & rectus KCA; ac propterea KC circulum CDE contingit. ducãtur CL FM ad ipsam AM perpendiculares. Si igitur manẽte KX semicirculi XCD GFN, & triãgula KCL KFM conuertãtur, quousque rursus restituantur in eundem locum, à quo moueri cœperunt, semicirculi quidem XCD GFN in sphæris ferentur; triangula vero KCL KFM conos efficient, quorum bases sunt circuli circa diametros CE FH, recti existẽtes ad KL axem, & eorum centra L M. coni uero sphęrarum contingent superficies, quoniam & KFC in omni conuersione semicirculos XCD GFN contingit. G H K L M

FED. COMMANDINVS.

Quod faciet sectiones in sphæris maximos circulos] *Ex primam propositione sphęricorum Theodosii.* A

Et anguli qui ad CF recti] *Ex 34. primi. Eucl. parallelogrãmorũ enim locorũ anguli, qui ex opposito ęquales sũt e tsũt recti qui ad A B anguli. ergo et qui ad C F recti erũt.* B

Ergo recta linea CF circulos CDE FGH contingit] *Ex 16 tertij libri elementorum.* C

Parallelogrammum vero AF cylindrum efficiet] *Ex 21 diffinitione vndecimi libri elementorum.* D

Quod

.

K[31,F], such that , as the radius of the circle CDE is to the radius of the circle FGH, so is AK to KB: let the point K be so taken, let KF be drawn tangent the circle FGH and let F&B joined. Then, through A, let AC be drawn parallel to BF, let C&F joined. Then since as AK is to KB so is AD to BN and AD is equal to AC and BN to BF, therefore, as AK is to KB, so AC to BF and AC is parallel indeed to BF. Therefore CFK is a straight-line[G], now the angle KFB is right[H], therefore the angle KCA is also right[K] and consequently KC touches the circle CDE[L], let CL & FM be drawn perpendicular to AM. If now, KX remaining fixed, the semicircles XCD & GFN and the triangles KCL & KFM be carried round and again restored to the position from wich they started, the semicircles XCD e GFN will move in coincidence with the spheres, the triangles KCL & KFM will generate cones[M] the bases of wich are the circles about the diameters CE & FH which are at right angles to KL axis, the centres of which being L & M and then the cones will touch the spheres along their surfaces, since KFC also touches the semicircles XCD & GFN throughout the wole rotation.

Federico Commandino

A. Will cut the spheres in great circles: *from first proposition of Teodosii Sphaerica.*

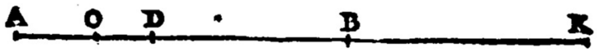
B. The angles at C & F will be right to CF: *from proposition 34 of first book of Euclid's Elements, in fact the opposite angles of the parallelograms are equal and since those to AB are right, then those to CF are right.*

C. So the straight line CF touches the circles CDE & FGH: *from 16° proposition of third book of Elements.*

D. The parallelogram AF will generate a cylinder: *from 21° definition of eleventh book of Elements.*

E Quod faciet sectiones in sphæris circulos] *Ex prima sphæricorum Theodosii.*

F Fieri igitur potest, vt sumatur aliquod punctum, velut K, ita ut H] *Illud autem punctum hoc modo inuenie mus. Ducatur seorsum ea, quae ex centro circuli maioris C*



DE, sitq; AD: & ex ipsa AD abscindatur AO aequalis ei, quae ex centro minoris circuli: fiatq; vt DO ad OA, ita AB ad aliam, quae sit BK. erit enim componendo, vt DA ad AO, hoc est ut quae ex centro circuli maioris ad eam quę ex centro minoris, ita AK ad KB.

G Recta igitur linea est CFK] *Hoc est si à puncto C ad K ducatur recta linea, transibit ea per F. quod nos demonstrauimus in commentarijs in decimam propositionem libri Archimedis de ijs, quae in aqua vehuntur, lemmate primo.*

H Sed angulus KFB rectus est,] *Ex 18 tertij elementorum.*

K Ergo & rectus KCA] *Ex 29 primi elementorum.*

L Ac propterea KC circulum CDE contingit] *Ex 17 tertij elementorum.*

M Triangula vero KCL KFM conos efficiunt] *Ex 18 diffinitione vndecimi libri elementorum.*

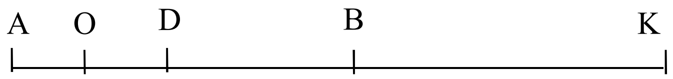
PROPOSITIO. II.

Si sphæra illuminetur à maiori sphęra, maior eius pars, quàm sit dimidia sphæra, il luminabitur.

Sphæra

E. This plane will cut the spheres in circles: *from the first proposition of Teodosii Sphaerica.*

F. It is possible to take a point as K: *we will find that point this way. Let be draw on the other side the radius of the greater circle*



CDE and let AD be said: *AO be carried on the same AD, equal to the radius of the lesser circle: and as DO be to OA so AB be to other which is BK. Componendo,*[32] *as DA is to AO, i.e. as the radius of greater circle is to the radius of lesser circle, so AK will are to KB.*

G. Therefore CFK is a straight line: *i.e. if from point C we draw a straight line to point K, this will carried through F. We have demonstrated this in commentary to tenth proposition of Archimed's book "De ijs quae in aqua vehuntur*[33]*",first lemma.*

H. Now the angle KFB is right: *from 18° proposition of third book of Elements*.

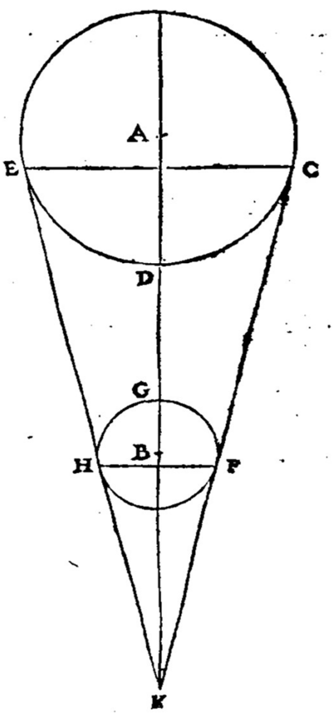
K. The angle KCA is also right: *from 29° proposition of first book of Elements.*

L. And consequently KC touches the circle CDE: *from 17° proposition of third book of Elements.*

M. The triangles KCL & KFM will generate cones: *from 18° definition of eleventh book of Elements.*

PROPOSITION II

If a sphere be illuminated by a sphere greater than itself, the illuminated portion of the former sphere will be greater than a hemisphere.

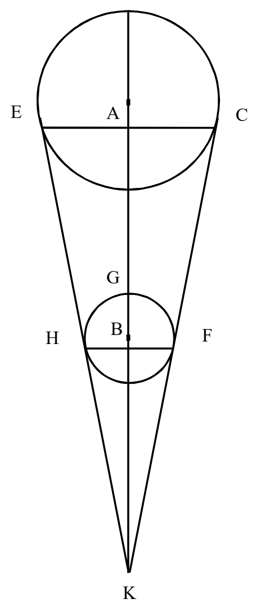


Sphæra enim, cuius centrum B à maiori ſphęra, cuius cētrū A illuminetur. Dico partem ſphęrę illuminatā, cuius cētrū B dimidia ſphæra maiorē eſſe. Q̄m enim duas inæquales ſphęras idem conus comprehendit, verticē habēs ad minorem ſphæram: ſit conus ſphæras comprehēdēs; & per axē planum producatur faciet illud ſectiones in ſphęris qui dē circulos, in cono autem triangulum. Itaq; faciat in ſphæris circulos CDE FGH; & in cono triāgulū CEK. manifeſtum eſt portionē ſphęrę, quæ eſt ad FGH circūferentiā, cuius baſis circulus circa diametrū FH, partē eſſe illuminatā à portione, quæ eſt ad circumferentiam CDE, cuius baſis circulus circa diametrū CE, rectus exiſtēs ad ipſam AB. etenim FGH circūferētia à circūferentia CDE illuminatur; quòd extremi radij ſunt CF EH: atque eſt in proportione FGH centrum ſphæræ B. Quare pars ſphęrę illuminata, dimidia ſphæra maior erit.

A

B

B FED.



Let a sphere, the center of which is B, be illuminated by a sphere greater than itself the centre of which is A. I say that the illuminated portion of the sphere, the centre of which is B, is greater than a hemisphere. Since two unequal spheres are comprehended by one and the same cone which has its vertex in the direction of the lesser sphere, let the cone comprehending the spheres be and let a plane be carried through the axis, this[A] plane will cut the spheres in circles and of course the cone in a triangle[B] so its will originate in the sphere the circles CDE FGH and in conus the triangle CEK. It is manifest that the portion of the sphere towards the circumference FGH, the base of which is the circle about the diameter FH, is the part illuminated by the portion of the sphere towards the circumference CDE, the base of wich is the circle about the diameter CE and at right angles to the straight line AB, in fact the circumference FGH is illuminated by the circumference CDE since CF & EH are the extreme rays: and B is the centre of the sfere within the arc FGH. So that the illuminated portion of the sphere is greater than a hemisfere.

FED. COMMANDINUS.

A Faciet illud sectiones in sphæris quidem circulos] *Ex 1. sphæricorum Theodosii. ut superius dictum est.*

B In cono autem triangulum] *Ex 3 propositione primi libri conicorum Apollonij.*

PROPOSITIO. III.

In luna minimus circulus determinat opacum, & splendidum, quando conus solem, & lunam comprehendens ad visum nostrũ verticem habeat.

Sit noster quidem visus ad A; solis centrum B; centrum vero lunæ, quando conus solem & lunam cõprehendens ad visum nostrum verticem habent, sit C: quando autem non habeat sit D. manifestum est puncta ACB in eadem recta linea esse. producatur per AB & D planum; quod faciet sectiones in sphę

A ris quidem circulos; in conis autem rectas lineas. faciat etiam in sphæra, per quam fertur centrum lu-

B næ circulum CD. ergo A est ipsius centrum; hoc enim ponitur. In sole autem faciat circulum EFR: & in luna quando conus solem, & lunam comprehendens ad visum nostrum verticem habeat, circulum HKL; quando autem non habeat, MNX. At in conis rectas lineas EA, AG, FO, OR: & axes AB

C BO. Quoniam igitur est, vt quæ ex centro circuli EFG ad eam, quæ ex centro circuli HKL, ita quæ ex cẽtro circuli EFG ad eam, quę ex cẽtro circuli MNX. Sed

A. This plane will cut the spheres in circles: from 1° book of *Teodosii Sphaerica,* as it has been said above.

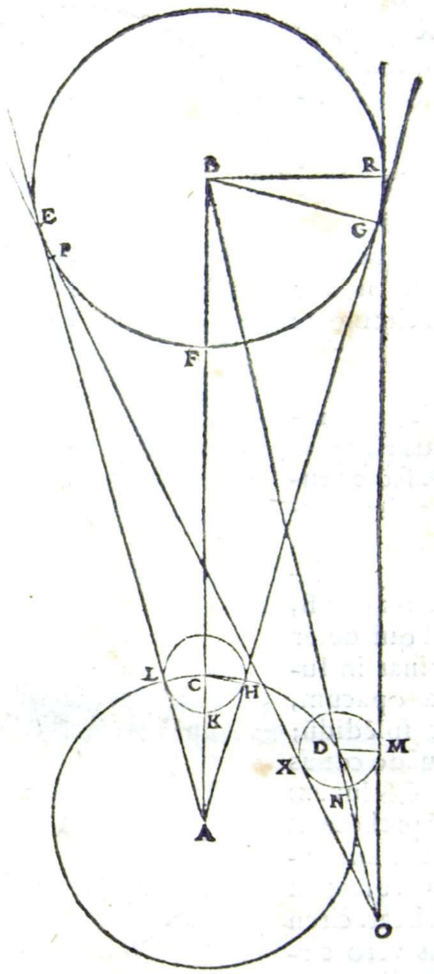
B. And of course the cone in a triangle: *from 3° proposition of the first book of Apollonius.*

PROPOSITION III

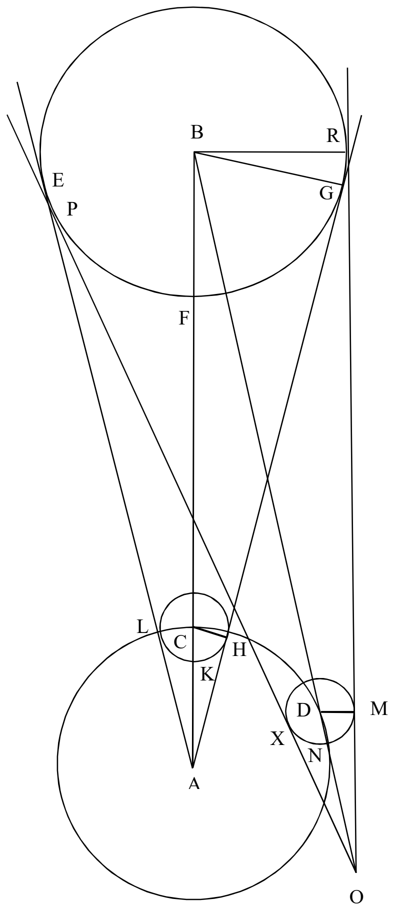
The circle in the moon which divides the dark an the bright portions is least when the cone comprehending both the sun and the moon has its vertex a tour eye.

For let our eye be at A, and let B be the centre of the sun, let C be the centre of the moon when the cone comprehending both the sun and the moon has its vertex at our eye, let D be the centre when this is not the case.It is then manifest that A,C,B are in a straight-line. Let a plane be carried through AB and D, this plane will cut the spheres in circles and the cones in straight-lines[A.]. This plane will produce also, in the sphera on wich the centre of the moon moves, the circle CD, therefore A is the centre of this circle, according to our hypothesis[B]. This plane will produce on the sun the circle EFR and on the moon the circle HKL, when the cone comprehending both the sun and the moon has its vertex at our eye, and in the circle MNX when this is not the case. This plane will also produce on the cones the straight-lines EA, AG, PO, OR and axes AB, BO. Since, as the radius of the circle EFG is to the radius of the circle HKL, so is the radius of the circle EFG to the radius of the circle MNX[C], but, as the

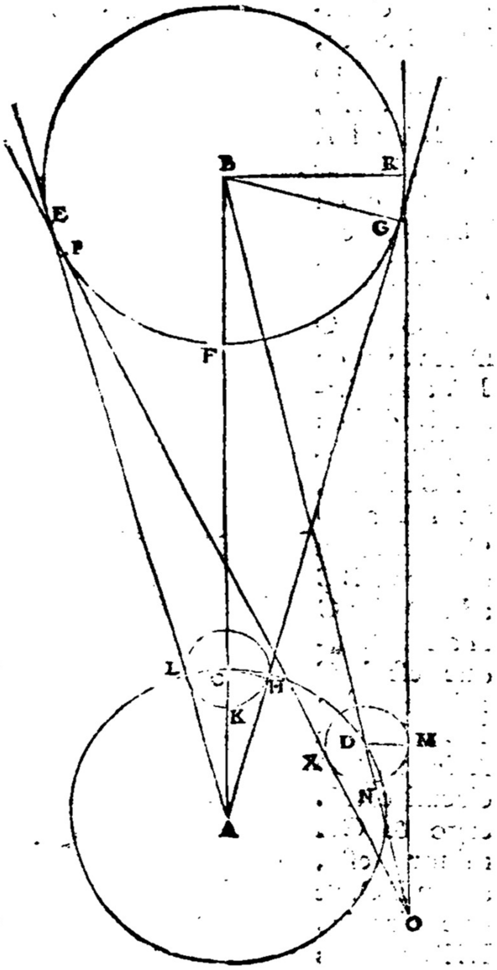
Sed vt quæ ex centro circuli EFG ad eã, quæ ex cẽtro circuli HKL, ita BA ad AC. vt aũt quæ ex cẽtro circuli EFG ad eã, quę ex centro circuli MNX, ita BO ad OD. & vt igitur BA ad AC, ita BO ad OD: & diuidendo vt BC ad CA, ita BD ad DO: permutandoque vt BC ad BD, ita CA ad DO. atque eſt BC minor quàm BD: eſt enim A ipſius CD circuli cẽtrũ. ergo & CA minor eſt, quàm DO. eſtque circulus HKL ęqualis circulo MNX



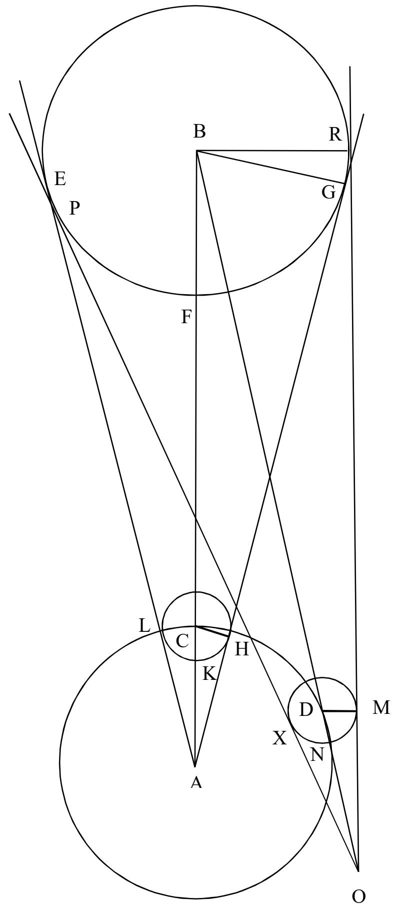
radius of the circle EFG is to the radius of the circle HKL, so is BA to AC[D]; and as the radius of the circle EFG is to the radius to the circle MNX, so is BO to OD, therefore as BA is to AC so is BO to OD[E] and, dividendo, as BC is to CA, so is BD to DO and also, permutando, as BC is to BD, so is CA to DO. And BC is less than BD[F]: for A is the centre of the circle CD, therefore CA i salso less than DO and the circle HKL is equal to the circle MNX



G minor igitur eſt HL, quàm MX propter lemma. Quare & circulus, qui circa diametrũ HL deſcribitur, rectus exiſtẽs ad ipsã AB minor eſt circulo deſcripto circa diametrũ MX, qui rectus eſt ad BO. ſed circulus circa diametrum HL, rectus exiſtens ad AB, eſt qui determinat in luna opacum, & ſplẽdidũ; quãdo conus ſolẽ, & lunam cõprehẽdens ad viſum noſtrũ verticem habeat. circulus vero circa diametrũ



therefore HL is also less than di MX for the Lemma[G]. Therefore the circle drawn about the diameter HL, at right angles to AB, is also less than the circle drawn about the diameter MX at right angles to OB. But the circle about the diameterHL at right angles to to AB is the circle which divides the dark a nd the bright portions in the moom, when the cone comprehending both the sun and the moon has its vertex ay our eye, instead the circle about the diameter



MX, rectus existens ad BO, in luna opacum, & splẽdidum determinat, quando conus solem, & lunã comprehendens verticem non habeat ad nostrum visũ. minor igitur circulus determinat in luna opacum, & splendidum, quando conus solem & lunam comprehendens ad visum nostrum verticẽ habeat.

FED. COMMANDINVS.

In conis autem rectas lineas] *Faciet enim triangula Ex 3. primi libri conicorum Apollonij.* A

Hoc enim ponitur] *Ex positione secunda huius. ponitur enim terram puncti, ac centri habere rationem ad sphęram lunę.* B

Quouiam igitur est vt quæ ex centro circuli EFG ad eam quæ ex centro circuli HKL, ita quę ex cẽtro circuli EFG ad eam, quæ ex centro circuli MNX] *Ex 7. quinti elemen. eadem ad aequales eandem habet proportionem.* C

Sed vt quæ ex centro circuli EFG ad eam, quæ ex centro circuli HKL, ita BA ad AC] *Iungatur enim CH & p B ipsi CH parallela ducatur BG. erit triangulũ ABG simile triangulo ACH. quare vt GB ad BA, ita HC ad CA ex 4. sexti: & permutando vt GB ad HC quae sunt ex centro circulorũ EFG HKL, ita BA ad AC. & similiter demõstrabitur, vt quae ex centro circuli EFG ad eam, quae ex centro circuli MNX, ita esse BO ad OD.* D

Et vt igitur BA ad AC, ita BQ ad OD] *Ex 11 quinti elementorum* E

Atque est BC minor, quàm BD] *Ex 8 tertij elementorum.* F

Minor igitur est & vt HL, quàm MX propter lemma.] *Vbi hoc lemma sit, nondum comperi, sed tamen illud* G

idem

MX, at right angles to BO, divides the dark and the bright portions in the moon when the cone comprehending both the sun and the moon has not its vertex a tour eye. Consequently the circle which divides the dark and the bright portions in the moon is less when the cone comprehending both the sun and the moon has its vertex at our eye.

Federico Commandino

A. And the cones in straight-lines: *this will indeed generate triangles: from 3° proposition of first book of Apollonius Conics.*

B. According to our hypothesis: *from second hypothesis of this book, indeed we have supposed that the earth is in the relation of a point and centre to the sphere in wich the moon moves.*

C. Since, as the radius of the circle EFG is to the radius of the circle HKL, so is the radius of the circle EFG to the radius of the circle MNX: *from 7° proposition of fifth book of Elements: a magnitude compared to equal magnitudes is in the same relationship.*

D. As the radius of the circle EFG is to the radius of the circle HKL, so is BA to AC: *indeed let be jointed C to H and let be draw through B the straing-line BG parallel to CH, the triangle ABG will be similar to the triangle ACH. As GB is to BA so HC is to CA for the 4° proposition of sixth book and permutando as GB is to HC, wich are the radii of the circles EFG and HKL, so BA is to HC, and similarly it is shown that as the radius of the circle EFG is to the radius of circle MNX, so BO is to OD.*

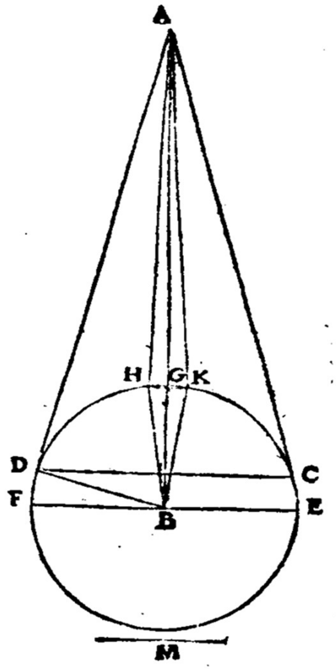
E. as BA is to AC so is BO to OD: *from 11° proposition of fifth book of Elements.*

F. And BC is less than BD: *from 8° proposition of third book of Elements.*

G. therefore HL is also less than di MX for the Lemma: *I have never verified where this lemma is stated, but however itself is* *idem in 24 propositione perſpectiuę Euclidis demonſtratur. Quoniam enim AC minor eſt, quàm OD, oculo poſito in A minus de corpore lunę cernetur, quàm eo poſito in O. ergo iũ ctis HL, MX, erit HL minor ipſa MX.*

PROPOSITIO. IIII.

Circulus in luna opacum, & ſplendidum determinãs non differt à maximo in ipſa circulo, quatenus ad ſenſum attinet.

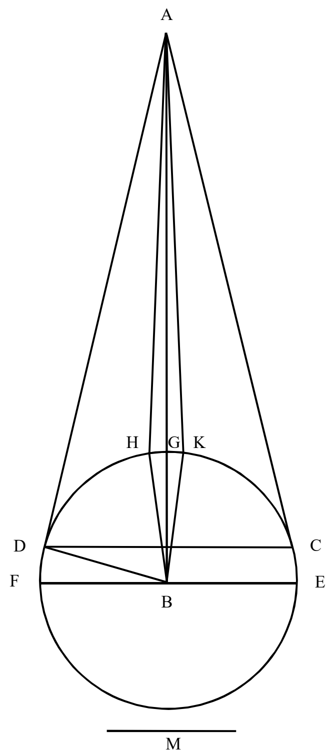


Sit noſter quidẽ viſus ad A, lunę vero centrum B; & iũ cta AB per ipſam planũ producatur, quod faciet ſectionem in ſphęra maximũ circulum. faciat circulum ECDF: & in cono rectas lineas AC AD DC. Circulus igitur circa diametrũ CD rectus exiſtẽs ad ipſam AB, eſt qui in luna opacũ, & ſpendidũ determinat. Dico eum non differre à maximo circulo, *proved in 24° proposition of Euclid's Optics. Since AC is also less than OD, when our eye is at A, we observed a less portion of the moon than when our eye is at O; also after having joined H to L and M to X, HL will be less than the same MX.*

PROPOSITION IV

The circle which divides in the moon the dark and the bright portions is not perceptiibly different from a great circle in the same moon.

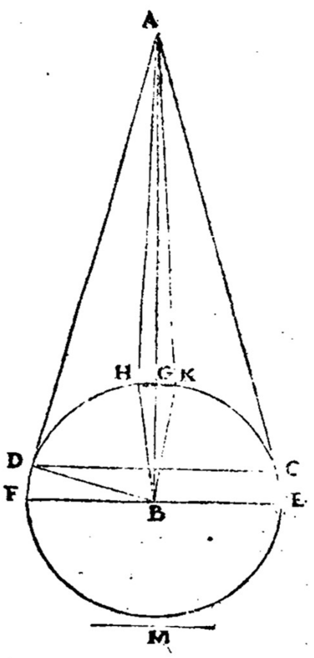
For let our eye at A and let B the centre of the moon. Let a plane be carried through joined AB, this plane will cut the sphere in a great circle ECDF and the cone in the straight-lines AC, AD, DC. Then the circle about the diameter CD, at right angles to AB, is the circle which divides the dark and the bright portions in the moon. I say that it is not perceptibliiy different from the great circle.



culo,quatenus ad ſenſum attinet.ducatur enim per B ipſi CD parallela EF ; & ponatur circumferentiæ DF dimidia vtraque ipſarum GK GH, & KB BH KA AH BD iungantur. Itaque quoniam poſitum eſt lunam ſubtendere quintamdecimam partem ſigni,angulus CAD conſiſtet in quintadecima ſigni parte.quinta decima autem ſigni pars,totius Zodiaci eſt pars centeſima,& octogeſima.quare CAD angulus in centeſima & octageſima parte totius Zodiaci conſiſtet, ideoque erit quattuor rectorũ pars cẽteſima & octogeſima; hoc eſt quadrageſima quinta pars vnius recti.eſtque eius dimidius BAD angulus.angulus igitur BAD eſt dimidij recti pars quadrageſima quinta. Et quoniam rectus eſt angulus A
ADB,habebit BAD angulus ad dimidiũ recti maiorem proportionem,quàm BD ad DA. quare BD B
minor eſt, quàm pars quadrageſima quinta ipſius DA;ac propterea BG ipſius BA multo minor erit, C
quàm quadrageſima quinta pars.& diuidendo BG ipſius GA minor,quàm pars quadrageſima quarta. D
ergo & BH multo minor eſt,quàm pars quadrageſima quarta ipſius HA.atque habet BH ad HA ma E
iorem proportionem, quàm angulus BAH ad ABH angulum,angulus igitur BAH anguli ABH minor eſt,quàm quadrageſima quarta pars. eſtque ipſius quidem BAH duplus angulus KAH; ipſius vero ABH duplus angulus KBH. ergo angulus KAH G
minor eſt,quàm quadrageſima quarta pars ipſius KBH. Sed angulus KBH eſt æqualis angulo DBF, H
hoc eſt angulo CDB,hoc eſt angulo BAD.angulus K L
igitur KAH anguli BAD minor eſt,quã quadrageſima quarta pars. At angulus BAD eſt quadrageſima quinta pars dimidij recti, hoc eſt vnius recti pars

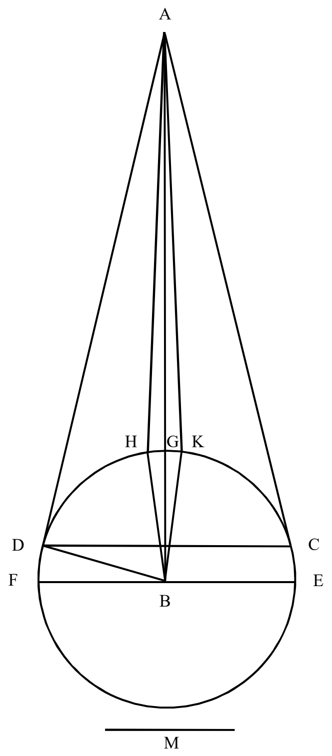
For let EF be drawn through B parallel to CD and GH & GK both be made half of arc DF. Let KB, BH, KA, AH, BD be joined. Then since, by hypothesis, the moon subtends a fifteenth part of a sign (of the zodiac), therefore the angle CAD stands on a fifteenth part of a sign. But a fifteenth part of a sign is a eightieth part of the wohle zodiac, so that the angle CAD stands on a eightieth of the whole zodiac, i.e.1/45th of a right angle, but the angle BAD is his half, therefore the angle BAD is 1/45th part of half a right angle and the angle ADB is right, the angle BAD has to half a right angle a ratio grater than that which BD has to DA[A], accordingly BD is less than 1/45th part of the same DA[B]; BG is much less than 1/45th part of the same BA[C] and, dividendo, BG is less than 1/44th part of the same GA, consequently BH is also much less than 1/44th of the same HA[D], and BH has to HA a ratio greater than that which the angle BAH has to the angle ABH[E]. Therefore the angle BAH is less than 1/44th part of the angle ABH[F], and the angle KAH certainly is double of the angle BAH, but the angle KBH is also double of the angle ABH, consequently the angle KAH is also less than 1/44th part of the same angle KBH[G]. But the angle KBH is equal to angle DBF[H], that is, to the angle CDB[K], that is to the angle BAD[L]. Therefore the angle KAH is less than 1/44th part of the angle BAD. But the angle BAD is 1/45th of half a right angle, i.e. 1/90th part pars nonageſima. ergo angulus KAH minor eſt, quàm recti pars 3960. magnitudo aũt ſpectata ſub tãtulo angulo inſẽſilis eſt noſtro viſui. atque eſt KH circumferẽtia ęqualis circumferẽtię DF. ergo DF noſtro viſui adhuc magis inſẽſilis eſt.

M ſi enim iungatur AF angulus FAD minor erit angulo KAH. quare punctũ D videbitur idem eſſe, quod F: & ſimili ratione C idem videbitur, quod E; ac propterea CD, quatenus ad ſenſum attinet non differt ab ipſa EF. circulus igitur determinans in luna opacum, & ſplẽdidum, quatenus ad ſenſum attinet à maximo circulo non differt.



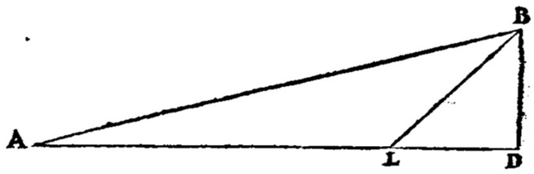
FED. COMMANDINVS.

A Et quoniam rectus eſt angulus ADB, habebit BAD angulus ad dimidium recti maiorem proportionem, quàm BD ad DA] *Deſcribatur ſeorſum triangulum* of a right angle, accordingly the angle KAH is less than 1/3960th part of a right angle, but of course a magnitude seen under such an so little angle is imperceptible to our eye. And the arc KH is equal to the arc DF; therefore DF is completely imperceptible to our eye. In fact, if A & F[M] be joined, the angle FAD will be less than the angle, KAH. Therefore the point D will seem to be the same with F. For the same reason C will also seem to be the same with E; consequently CD is not perceptibly different from the same EF. Therefore the circle which divides the dark and the bright portions in the moon is not perceptilly different from a great circle.



Federico Commandino

A. and the angle ADB is right, the angle BAD has to half a right angle a ratio grater than that which BD has to DA*: let be drawn in another part the triangle ABD and let part, on the same DA,* lum ADB,& ab ipsa DA abscindatur DL aequalis DB, & BL iungatur. erunt trianguli BLD, anguli DBL DLB inter se aequales. & cum angulus ad D sit rectus, vterque ipsorũ recti dimidius erit. Itaque duo triangula rectangula sunt A 5. primi. 32. primi.



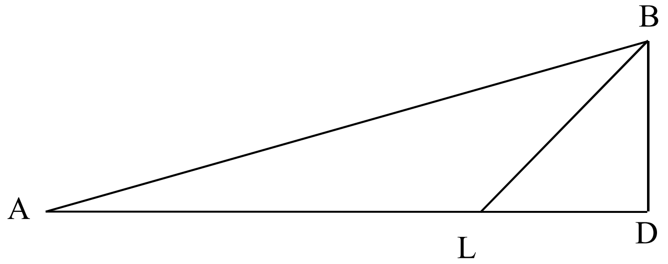
BD, LBD, quorum anguli ad D recti, trianguli vero ABD latus BD est commune triangulo LDB, & latus AB maius latere LB. ergo ex ijs, quae nos demonstrauimus in commentarijs in librum Archimedis de numero arenę, angulus BLD ad angulũ BAD maiorẽ quidem proportionẽ habet, quàm B A latus ad latus BL, minorem vero, quàm latus AD ad latus DL. quare conuertendo ex 26 quinti elementorum, quã nos addidimus ex Pappo, angulus BAD ad angulum BLD, hoc est ad dimidium recti maiorem proportionem habet, quã latus DL, hoc est BD ipsi aequale, ad latus DA. 7. quinti.

Quare BD minor est, quàm pars quadragesima quinta ipsius DA] Sit enim, vt angulus BAD ad dimidiũ recti, ita quępiã recta linea, in qua M ad ipsam DA, erit M quadragesima quinta pars ipsius DA, & habebit ad DA maiorem proportionem, quàm BD ad DA. ergo BD minor est, quàm M; ac propterea minor, quàm pars quadragesima quinta ipsius DA. B 8. 10. quinti.

Ac propterea BG ipsius BA multo minor erit, quàm quadragesima quinta pars] Est enim BG aequalis ipsi BD, & BA maior quàm AD, cum maiori angulo subtendatur. C

Ergo BH multo minor est, quàm pars quadra- D

DL equal to DB and let be joined B & L; the angles DBL and DLB of triangle BLD will be equal to each other and since the angle D



is right, both of them will be half right angle and so ABD & LBD are two triangles at right angles in D; then the side BD of triangle ABD is common to the triangle LDB, and the side AB is greater than the side LB, so, for what we have shown in the commentaries to the Archimedes book Sand Reckoner, the angle BLD has to the angle BAD a ratio certainly grater than that wich the side BA has to the side BL, however a ratio less than that wich the side AD to the side DL. Therefore, invertendo, from 26° proposition of fifth book of Elements, that we have added from Pappus, the angle BAD has a ratio to the angle BLD, that is half right angle, greater than that the side DL, equal to BD, to DA.[7° prop. of fifth b.]

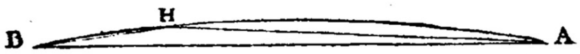
B. Accordingly BD is less than 1/45th part of the same DA: i*ndeed, as the angle on BAD is to half right angle, so will be a some straight-line, in wich M will be 1/45th DA, to same DA, and will have a ratio to DA grater than BD to DA. Then BD*[10° prop. of fith b.] *is less than M and therefore less than 1/45 part of the same DA .*

C. BG is much less than 1/45th part of the same BA: *Indeed BG is equal to BD, and BA is greater than AD, considering that BA is subtended by a greater angle.*

D. Consequently BH is also much less than 1/44th of the same HA:

gesima quarta ipsius HA.] *Nam BH est aequalis ipsi B G; HA vero maior, quàm GA, ex 8 tertij elemen.*

E Atque habet BH ad HA maiorem proportionẽ, quàm angulus BAH ad ABH angulum] *Describa-*



tur circa triangulum ABH circulus AHB, habebit recta linea AH ad rectam HB minorem proportionem, quàm circumferentia AH ad HB circumferentiam, ex demonstratis à Ptolemęo in principio magnę constructionis. vt autem circumferentia AH ad circumferentiã HB, ita angulus ABH ad BAH angulum. recta igitur linea AH ad rectam HB minorem habet proportionem, quàm angulus ABH ad angulũ BAH. quare conuertendò ex 26 quinti, recta linea BH ad rectã HA maiorem proportionem habebit, quàm angulus BAH ad ABH angulum.

Vlt. sexti. / 11. quinti.

F Angulus igitur BAH anguli ABH minor est, quàm quadragesima quarta pars] *Immo vero multo minor.*

G Ergo angulus KAH minor est, quàm quadragesima quarta pars ipsius KBH] *Ex 15 quinti elemen.*

H Sed angulus KBH est æqualis angulo DBF] *Ita enim ponitur.*

K Hoc est angulo CDB] *Ex 29 primi elementorum.*

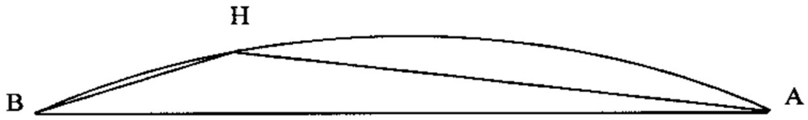
L Hoc est angulo BAD] *Ex 8 sexti elementorum.*

M Si enim iungatur BF, angulus FAD minor erit angulo KAH]

PAPPVS IN EODEM LOCO.

Describemus autem vnum lemma ex ijs, quae traduntur in *indeed BH is equal to BG; then HA is greather than GA from 8° proposition of tirth book of Elements.*

E. BH has to HA a ratio greater than that which the angle BAH has to the angle ABH: *We drawn the circle*



AHB around the triangle ABH , the straight-line AH will have to the straiht-line HB a ratio less than the circle AH to the circle HB, has Ptolemy proved at the beginning of Liber Magnæ Costructionis. As then the arc AH is to the arc HB, so theangle ABH is to the angle BAH[last of 6° b.]. *The straight-line AH has to the straight-line HB a ratio less, than that wich has to the angle ABH to the angle BAH* [*31 of 5° b*]. *Consequently, convertendo, in according to proposition 26 of fifth book, the straight-line BH has a ratio to the straight-line HA less than that wich the angle BAH has to angle ABH.*

F. Therefore the angle BAH is less than $1/44^{th}$ part of the angle ABH: *certainly minovery less*.

G. Consequently the angle KAH is also less than $1/44^{th}$ part of the same angle KBH: *from 15° proposition of fifth book of Elements*.

H. But the angle KBH is equal to angle DBF: *so in fact by hypothesis.*

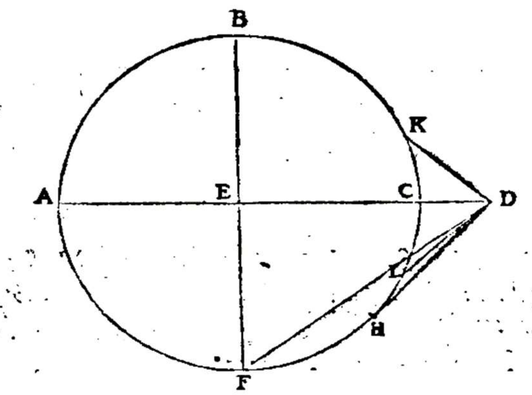
K. That is, to the angle CDB: *from 29° proposition of first book of Elements.*

L. that is to the angle BAD: *from 8° proposition of sixth book of Elements.* If we join A[34] with F, the angle FAD will be less than the angle KAH.

Pappus in the same place

We now describe a lemma of those mentioned

In quartum theorema eiusdem libri, inquisitione dignum.

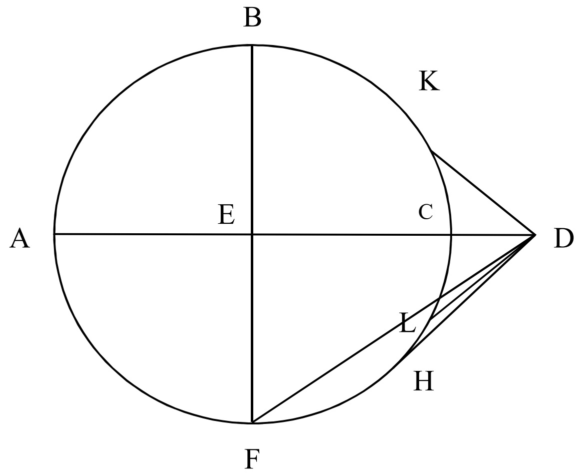


Sit circulus ABC, cuius diameter producta AC D; centrum E: & à puncto E ipsi ACD ad rectos an gulos ducatur BEF: ab ipso autem D dncatur DH, circulum ABC contingens: & dimidiæ ipsius FH æqualis ponatur ad vtrasque partes C, videlicet K C CL: iunganturque AD DL FD. Dico angulum KDL angulo FDH maiorem esse. *Pręmittuntur autem hęc.*

Sit circulus ABC, cuius diameter producta AC D: & à puncto D ducatur quæpiam recta linea DE F. Dico circumferentiam AF circumferentia CE maiorem esse.

Sumatur enim circuli centum G: & GF GE iungantur. erit angulus ad F angulo ad E ęqualis. Et quoniam triangulum est GFD, & angulus exterior AGF maior est interior, & opposito, eo, qui ad F; hoc est eo, qui ad E; angulus au- 5. primi.

by fourth theorem, worthy of being examined, of the same book,.

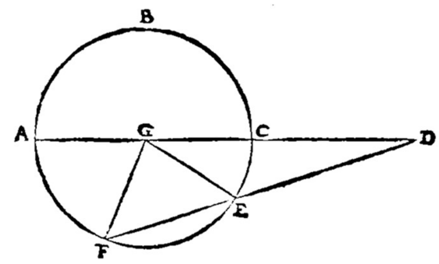


For let be described the circle ABC, let produced its diameter ACD; let E be its center, and let be drawn BEF from point E at right angles to ACD; let be produce from D DH what touches the circle ABC, and an arc equal to FH is located on either side of C, that are KC e CL and let be joined A & D, D & L, F & D. I say that the angle KDL is greather than the angle FDH.

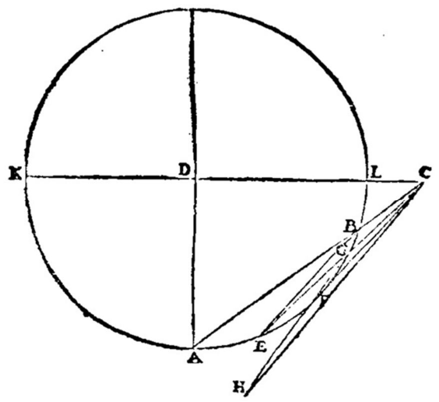
Let say before this.

Let be drawn the circle ABC whose produced diameter is ACD; from point D let be drawn any right-line DEF. I say that the arc AF is greater than arc CE.

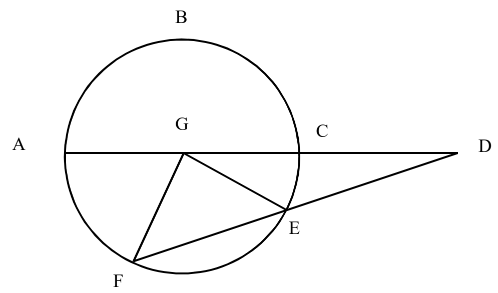
Indeed let assume the center of the circle G: let be joined also G & F and G & E. The angle to F will be equal angole in E[5° Prop. of first b.]*. Since GFD is a triangle, then the exterior angle AGF is greather than he interior and also opposite which is in F; i.e. at E; but The angle in E is greater*



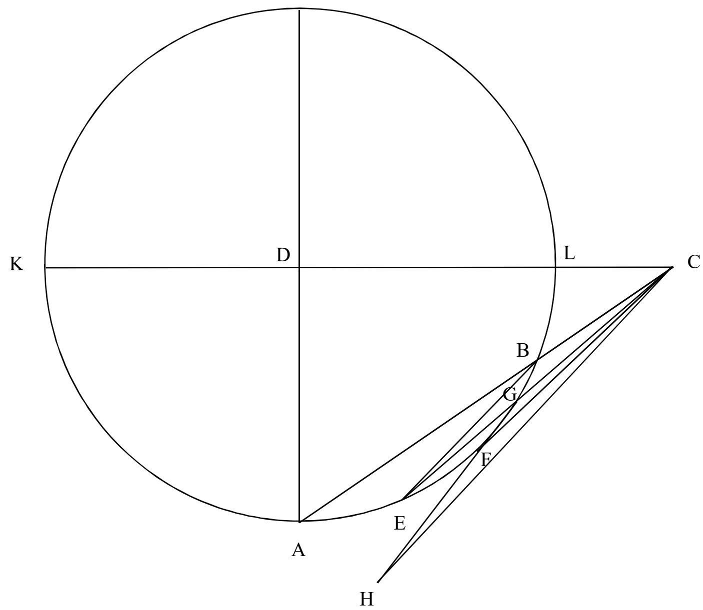
tem ad E maior est angulo DGE, propterea quòd est extra triangulum: erit angulus AGF angulo EGD maior. & sunt ad centrum. circumferētia igitur AF maior est circumferētia CE. quod demōstrare oportebat.



Sit circulus AB, cuius centrum D; & extra circulum punctum C: ducanturq; CDK, & circulum con tin-



than the angle GDE, since the same is exterior to the triangle: the angle AGF will be greater than the angle EGD. But these angle are also central angles. The arc AF is then greather than arc CE, as it was necessary to demonstrate.



Let be take into consideration the circle AB with center in D and let be C a point esterior to the circle and let be drawn CDK and also CF what touches tingens CF. deinde per D centrum ad rectos angulos ipsi KL diametro agatur DA; seceturque AF circumferentia bifariam in puncto E. & CBA CGE iungantur. Dico angulum ACE angulo ECF maiorem esse.

Iungantur enim EB FG. & quoniam EB maior est, quã
FG, & BC minor, quàm CG; habebit EB ad BC maiorẽ pro- 8. quinti.
portionem, quàm FG ad GC. Itaque fiat vt EB ad BC, ita H
G ad GC, & HC iungatur. Quoniam igitur anguli ABE EG 21. tertij.
F inter se aequales sunt, quòd & circumferentia AE circũ-
ferentię EF; & reliqui anguli EBC FGC aequales; & cir- 13. primi.
ca aequales angulos latera sunt proportionalia: erit triãglũ 6. sexti.
EBC triãgulo HGC ęquiãgulũ. ergo anguli ACE ECH iter
se ęquales sunt. angulus igitur ACE angulo ECF est maior.

Sit deniq; eadem figura, quę prius; & eadem maneant. Dico angulũ KDL angulo FDH maiorẽ esse.

Secetur circumferentia FH bifariam in puncto M, & iun
gatur MD. constat igitur ex eo, quod proxime ostensum est,
angulum FDM maiorem esse angulo MDH. producantur F
EB DL ad puncta NX: sitq; ipsi AD aequalis NF, & NM,
ND iungantur. Itaque quoniam circulus est ABC, cuius dia-
meter producta ACD, & à puncto D acta est DLX ad con- 39. huius.
cauam circumferentiam; erit circumferẽtia AX maior, quã
circumferẽtia CL. sed CL est aequalis FM; vtraque enim est
circumferentiae FH dimidia. circumferentia igtur AX ma-
ior est, quàm FM. ponatur ipsi FM aequalis cirumferentia
AO; iunganturq; AO OD. Et quoniam cirumferentia AF
C semicirculi aequalis est circumferentiae semicirculi FCB,
quarum AO est aequalis MF; erit & reliqua OC reli-
quae MB aequalis. sed circumferentiae quidem OC in-
sistit DAO angulus; circumferentiae vero MB insistit 27. tertij.
angulus NFM. ergo angulus DAO est aequalis angu- 31. tertij.
lo NFM. atque est vterque eorum recto minor. & cum

AD

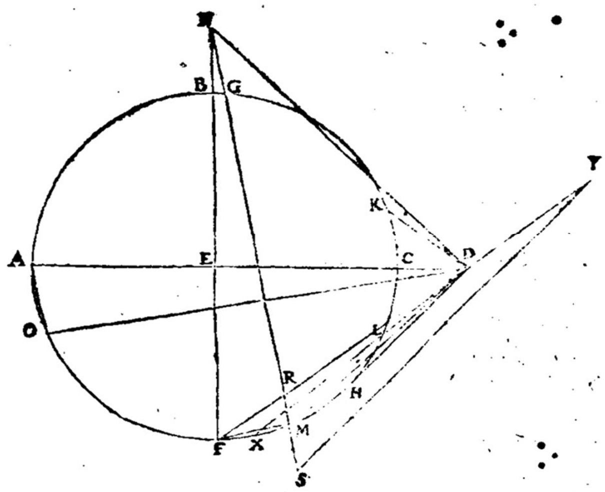
the circle. After let be drawn DA through the center D, at right angles to same diameter KL, and let the arc AF be cut in two parts in point E. Let be also joined CBA & CGE.

I say that angles ACE is greater than the angle ECF.

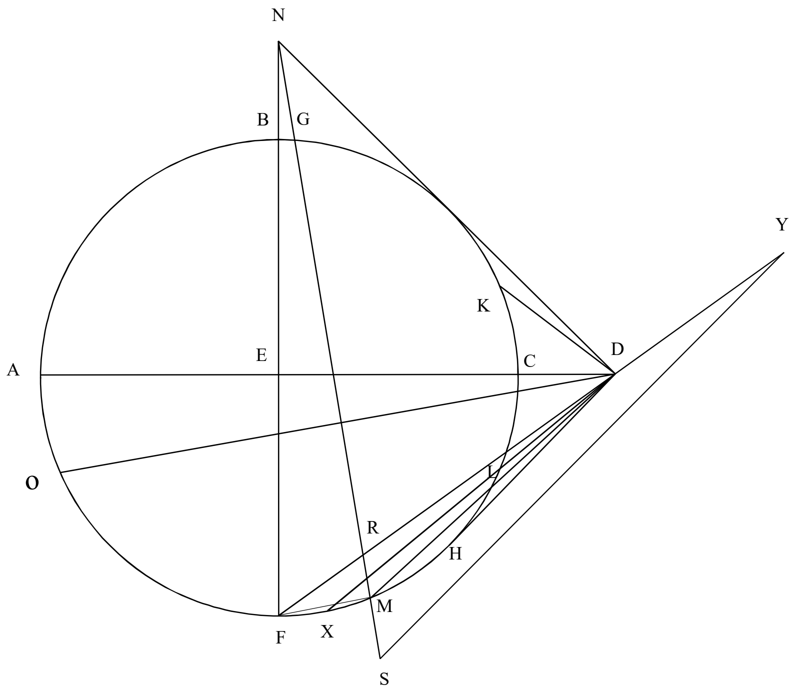
Let be joined indeed EB & FG and since EB is greater than FG, and BC is less than CG, EB will have a ratio to BC greater than tath FG have to GC.[3° prop. of fift b.] *So as EB is to BC, so HG is to GC.*[21° prop. of tirth b.] *Let be joined also H & C. Therefore so the angles ABE and EGF are equal to each other, since the arc AE is equal to arc EF*[13° prop. o ffirst b.]*, also the remaining angles EBC and FGC are equals,also the correspondent sides to equal angles are proportional, the triangle EBC will be equiangular with the triangle HGC.*[6° prop. del sesto] *Therefore the angles ACE and ECH are equal to each other. The angle ACE is consequently greater than the angle ECF.*

We considered at last the same foregoing image, unchanged. I say that the angle KDL is greater than angle FDH.

We cut the arc FH in two parts at point M and we joint M & D. He is evidently, for what it has been shown just now, that the angle FDM is greather than the angle MDH. Let be produce FEB and DL at points N and X, and let NF be equal indeed to AD,let be joined also N & M and N & D. Therefore since ABC is the circle, of which the ACD diameter has been produced and DLX has been drawn from point D towards the circle concavity,[39° prop. of this] *the arc AX will be greater maggiore than arc CL; but CL is equal to FM, both in fact are half arc FH; therefore the arc AX is greater than FM. Let the arc AO indeed be equal to MF and let be joined A & O and O & D; since the semicircleAFC is equal to the semicircle FCB, of wich AO is equal to MF, also the remaining OC will be equal to remaining MB, but the angle DAO howevwr insist in the arc OC, as also the angle NFM insist in the arc MB*[27° prop.ofl tirth b.] *,then the angle DAO is equal to the angle NFM,*[31° prop. of tirth b.]*but both are less than a right angle retto, and since*



AD sit aequalis FN, & DO ipsi FM, duae DA AO
duabus NF FM aequales sunt; & angulus DAO est aequa
4. primi. *lis angulo NFM: quare & basis OD basi NM, & reliqui an*
guli reliquis angulis sunt aequales. angulus igitur ADO est
aequalis angulo FNM. Rursus quoniã semicirculi circũferẽ
tia est FAB, erit FABG semicirculo maior, cui insistit angu
31. tertii. *lus FMG. ergo FMG maior est recto; & ipsi subtẽditur re-*
cta linea FR. angulo aũt acuto RFM subtẽditur RM. quare
19. primi. *FR maior est, quàm RM. Itaque producatur RM ad S; & ip*
si FR aequalis ponatur RS. Et quoniam tota ACD aequalis
est toti FBN, quarum AE est aequalis EF; erit reliqua ED
5. primi. *ipsi EN aequalis: ideoq; angulus EDN est aequalis angulo E*
ND; & ADN maior angulo DNR. quare latus NR latere R
D est maius. producatur RD ad Y: ponaturq; ipsi NR aequa
lis



AD is equal to FN, and AO[35] is indeed equal to FM, both DA and AO are equal to both NF and FM, and the angle DAO is equal to the angle NFM, consequently both the base OD is equal to the base NM, and the remaining angles are equal to the remaining angles, then the angle ADO is equal to the angle FNM.

Again since FAB is a arc of semicircle, FABG will be greater than the semicircle, on wich stands the angle FMG, then FMG is greater than a right angle,[3°prop. of tirth b.] but the right-line FR subtend the same angle, while RM subtend the acute angle RFM, therefore FR is greater than RM.[19° prop. of first b.] Then let be produce RM to S, and let be RS just equal to FR. Since the whole ACD is equal to wole FBN, of wich AE is equal to EF, the remaining ED will be equal just to EN; [5°prop. of firts b.] therefore the angle EDN is equal to the angle END and ADN is greater than the angle DNR; therefore the side NR is greater than the side RD; let be produce RD to Y and suppose RY equal just to NR and let joined S & Y.

lis RT; & ST iungatur. Quoniam igitur FR est aequalis RS, & NR ipsi RT; duę FR RN duabus SR RT aequales sunt: & angulus FRN aequalis angulo SRT, quòd sint ad verticem. ergo & basis NF basi ST; & reliqui anguli reliquis angulis aequales. quare angulus RFN est aequalis angulo RST. sed angulus RMD maior est angulo RST, cum sit extra triangulum. angulus igitur RMD angulo RFN est maior. est autem & FRN angulus aequalis angulo MRD. quare & reliquus FNR maior reliquo RDM. At ostensum est angulum FNR angulo ADO esse aequalem. angulus igitur ADO angulo RDM est maior; ac propterea ADX angulus multo maior est angulo RDM. anguli autem ADX duplus est angulus KDL: et anguli RDM minor, quàm duplus ostensus est angulus FDH. ergo KDL angulus angulo FDH maior erit. 4. primi. In antecedente.

PROPOSITIO. V.

Cum luna dimidiata nobis apparet, tunc maximus circulus, qui est iuxta determinantem in luna opacum, & splendidum, in visum nostrum vergit: hoc est maximus circulus, qui est iuxta determinantem, & noster visus in vno sunt plano.

Luna enim dimidiata existente, apparet circulus determinans opacum, & splendidum ipsius, vergere in nostrum visum: & ab eo non differt circulus maximus, qui est iuxta determinantem. cum igitur luna dimidiata nobis apparet, tunc circulus maximus, qui est iuxta determinãtem, in visum nostrum vergit. 9. positione. 4. huius.

Since then FR is equal to RS, and NR is equal just to RY, both FR ed RN are equal to the two SR and RY, and the angle FRN is equal to the angle SRY because are at the vertex, then also the base NF is equal to the base SY, and the remaining angles are equal to the remaining angles,[4° prop. of first b.] *therefore the angle RFN is equal to the angle RSY, but the angle RMD is greater than the angle RSY, because is exterior to triangle, then the angle RMD is greater than the angle RFN, and also the angle FRN is equal to the angle MRD, consequently the remaining FNR is greater than the remaining RDM.*
Moreover it was proved that the angle FNR is equal to the angle ADO, then the angle ADO is greater than the angle RDM, therefore the angle ADX is greater than the angle RDM, the angle KDL is also twice the angle ADX and we have shown that the angle FDH is less than twice of the angle RDM; therefore the angle KDL will be greater to the angle FDH.

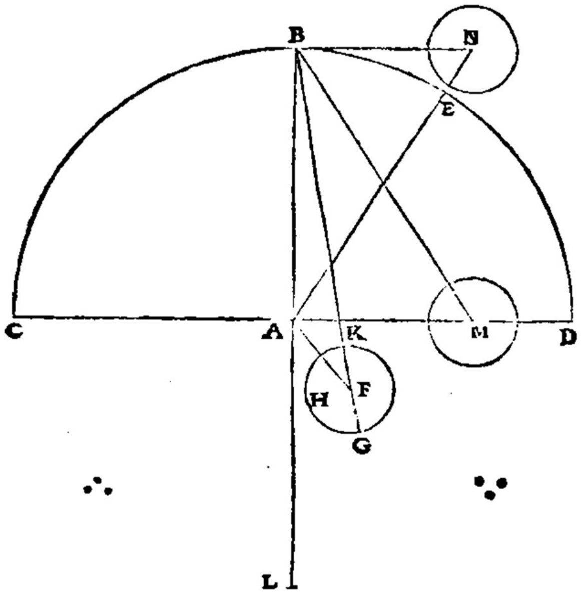
PROPOSITION V

~~When~~ the moon appears to us halved, the great circle, wich is very near the circle which divides the dark and the bright portions in the moon, is then in the direction of our eye; that is to say, the great circle, what is very near the dividing circle, and our eye are in one plane.

When the moon is halved, the circle which divides the bright and the dark portions of the moon is in the direction of our eye;[3° hypotesis] but the great circle is indistinguishable from that,[4° hypothesis] what is very near the dividing circle, therefore, when the moon appears us halved, the great circle very near to the dividing circle is then in the direction of our eye.

PROPOSITIO. VI.

Luna infra ſolem fertur, et dimidiata exiſtens à ſole minus quadrante diſtat.

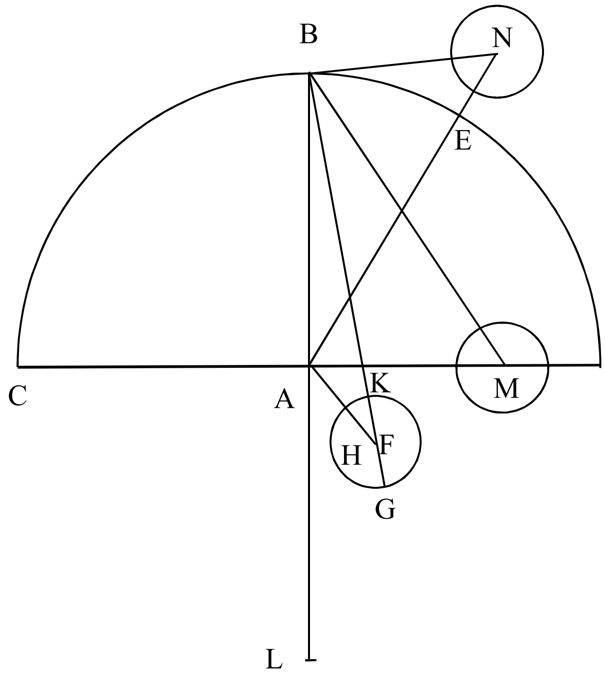


Sit enim noſter viſus ad A, ſolis autem centrum B: & iuncta AB, per ipſam, & per centrum lunæ dimidiatæ exiſtentis planum producatur. faciet vtiq; ſectionem in ſphæra, per quam fertur centrum ſolis circulum maximum. faciat circulum CBD: & à pũcto A ipſi AB ad rectos angulos ducatur CAD.

qua

PROPOSITION VI

The moon moves lower[36] *than the sun, and, when it is halved, is distant less than the quadrant from the sun.*



Let our eye be at A, and let B be the centre of the sunA; let A & B joined and let a plane be carried through right-line AB and the centre of the halved moon. This plane certainly will cut in a great circle the sphere on which the centre of the sun moves: let the circle CBD be done; and from point A let CAD be drawn at right angles to same AB, quadratis igitur est circumferentia BD. Dico lunã infra solem ferri, & cum dimidiata existat, minus quadrãte à sole distare: hoc est centrum ipsius intra rectas lineas BA AD, & circumferentiam DEB cõtineri. Si enim non, sit centrum ipsius F intra rectas lineas DA AL, & BF iungatur. erit BF axis coni solem, & lunam compræhendentis: atque erit perpendicularis ad maximum circulũ, qui in luna opacum, & splendidum determinat. Sit igitur maximus circulus in luna iuxta determinãtem opacum & splendidum GHK. Et quoniam luna dimidiata existente maximus circulus, iuxta determinantem in luna opacum & splendidum, & noster visus sunt in vno plano, iungatur AF. ergo AF est in plano circuli KGH: est autem & BF circulo KGH ad rectos angulos. quare & ipsi AF, ac propterea angulus BFA rectus est. Sed & obtusus est angulus BAF. quod fieri non potest. non igitur punctum F est in loco intra angulum DAL contento. Dico neque esse in ipsa AD. SI enim fieri potest, sit M: & rursus BM iũgatur: sitq; maximus circulus iuxta determinantẽ, cuius centum M. Eadem ratione ostendetur angulus BMA rectus esse ad maximum circulum. sed & BAM est rectus. quod fieri non potest. non igitur in ipsa AD est centrum lunæ dimidiatę existentis. ergo erit intra rectas lineas BA AD. Dico prætere a esse intra circumferentiam BED. Nam si fieri potest, sit extra in puncto N; & eadem construantur. ostendemus angulum BNA rectum esse. maior igitur est BA, quàm AN. sed BA est æqualis AE. ergo & AE, quàm AN maior erit. quod fieri nõ potest. non igitur centrum lunę dimidiatę existentis est extra circumferẽtiam BED. similiter ostende-

A (margin, line 7) · B (line 12) · C (line 17) · D (line 18)

D tur

Then the arc BD is that of a quadrant. I say that the moon moves lower than the sun, and, when halved is distant less than a quadrant from the sun: that is to say is center is contaibed between the straight-lines BA, AD and the arc DEB. Let us suppose that it is not, let its centre of the same F between the straight-lines DA & AL and let also B & F be joined, then will BF be the axis of the cone which comprehends both the sun and the moon and will be at right angles to the great circle which divides the dark and the bright portions in the moon[A]. Indeed let be GHK the great circle which divides the dark and the bright portions in the moon. Then since, when the moon is halved, the great circle, just by the circle which divide the dark and the bright portions in the moon, and our eye are in one plane[B], let A&F be joined. Therefore AF is in the plane of the circle KGH; but then also BF is at right angles to the circle KGH, therefore also to AF, and for this reason the angle BFA is right,[C] but the angle BAF is obtuse[D], which is impossible. Therefore the point F is contained in the space into the angle DAL. I say that is it not even on the same AD. In fact we suppose it be point M: and again let B&M be joined; and let the great circle be just by to the dividing circle, its centre being M. Then, whith the same reasoning, it can be shown that the angle BMA is right to the great circle, but the angle BAM is right, which is impossible. Therefore the centre of the moon, when halved, is not on AD, therefore it is between the right lines BA and AD. Moreover I say that is also within the arc BED: let us suppose in fact that it be, outside, at point N, and let the same constructions be made, we can proved that the angle BNA is right, therefore BA is greater than AN: which is impossible. Therefore the centre of the moon, when halved, is not outside the arc BED. Similary it can be proved that neither it placed tur neque esse in ipsa BED circumferentia. ergo in tra ipsam sit necesse est. luna igitur infra solem fertur, & dimidiata existens minus quadrante à sole distat.

FED. COMMANDINVS.

A Erit BF axis coni solem,& lunam comprehendẽtis: atque erit perpendicularis ad maximum circulum, qui in luna opacum, & splendidum determinat] *Ex demonstratis in tertia propositione huius.*

B Et quoniam luna dimidiata existente maximus circulus iuxta determinantem in luna opacum & splendidum, & noster visus in vno sunt plano] *Ex antecedente.*

C Quare & ipsi AF, ac propterea angulus FBA rectus est] *Ex tertia diffinitione vndecimi elementorum.*

D Sed & obtusus est angulus BAF. quod fieri non potest] *Essent enim trianguli ABF tres anguli maiores duobus rectis.*

PROPOSITIO VII.

Distantia, qua sol à terra distat, distantię qua luna distat à terra maior quidem est, quàm duodeuigintupla, minor uero, quàm vigintupla.

Sit solis quidem centrum A; terræ vero centrum B. & iuncta AB producatur. lunæ autem dimidiatæ existentis centrum sit C: & per AB, & C planũ producatur, quod faciat sectionem in sphæra, per quam fertur on the arc BED itself, therefore it needs be within itself, The moon moves lower than the sun, and, when it is halved, is distant less than the quadrant from the sun.

Federico Commandino

A. BF be the axis of the cone which comprehends both the sun and the moon and will be at right angles to the great circle which divides the dark and the bright portions in the moon: *from the demonstration contained in tirth proposition of this book.*

B. Since, when the moon is halved, the great circle, just by the circle which divide the dark and the bright portions in the moon, and our eye are in one plane*: from antecedent.*

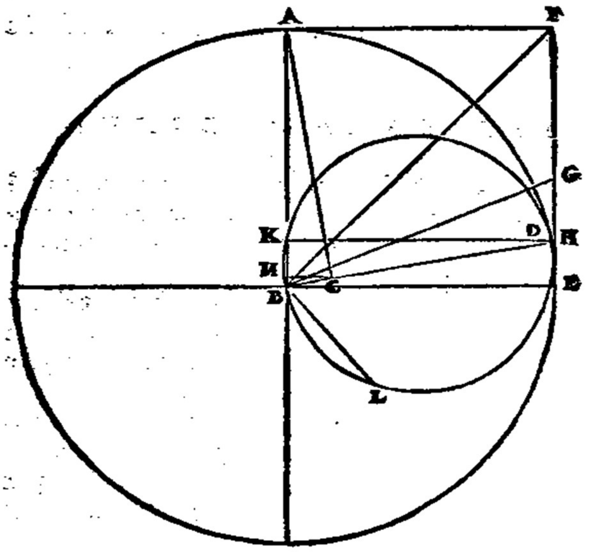
C. For this reason the angle BFA is right: *from tirth definition of eleventh book of Elements.*

D. But the angle BAF is obtuse: *indeed the three angles of triangle ABF would be greater tha two right angles.*

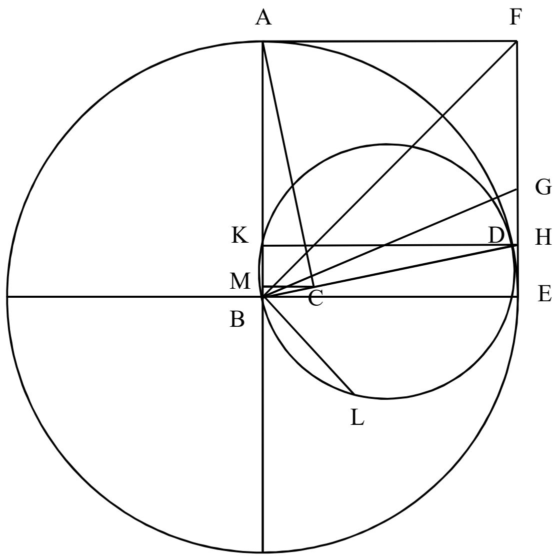
PROPOSITION VII

The distance that separates the sun from the earth is greater than eighteen times, but less than twenty times, the distance of the moon from the earth.[37]

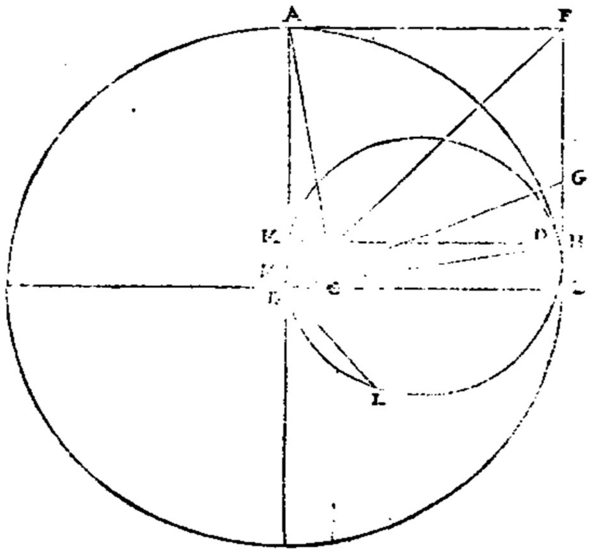
Let A be the centre of the sun, whilst B that of the earth B, let A& B be joined and produced; let C be the centre of the moon when halved, let a plane be carried through AB and C which cut the sphere and let be the great circle ADE thath on which the centre of the sun moves, and let A&C and C&B be joined and let BC be produced to D.



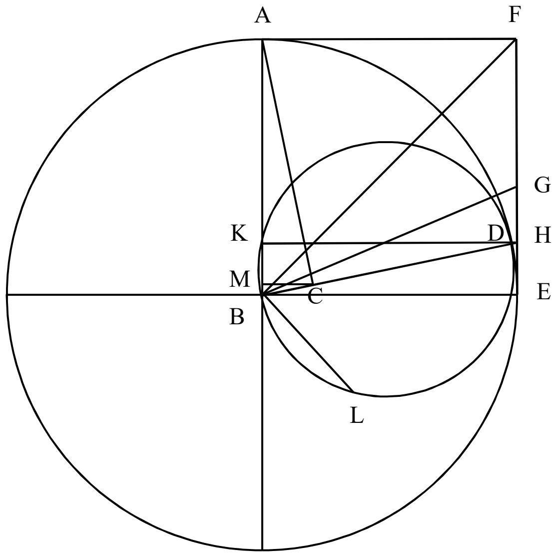
fertur centrum solis, maximum circulu m ADE, &
AC CB iungatur: producaturq; BC in D. erit vti- 3. huius.
que angulus ACB rectus, propterea quòd pũctum
C sit lunę dimidiatæ centrum. ducatur à puncto B
ipsi BA ad rectos angulos BE. ergo circumferentia A
ED erit trigesima pars circumferentiæ EDA. posi-
tum est enim, cum luna dimidiata nobis apparet, di
stare eam à sole minus quadrante, quadrantis parte
trigesima. quare & EBC angulus est trigesima pars
vnius recti. compleatur parallelogrammum AE: & B
BF iungatur. erit angulus FBE recti dimidius. sece-



The angle ACB will be certainly right, because the point C is the centre of the moon when halved; let BE be drawn from B at right angles to BA itself, then the arc ED will be one-thirtieth part of arc EDA[A]; indeed, by hypothesis, when the moon appears to us halved, its distance from the sun is less than a quadrant by one-thirtieth of a quadrant, therefore the angle BC is also one-thirtieth part of a right angle. Let the parallelogram AE be completed and let B&F be joined[B], the angle FBE will be half a right angle. Let the angle FBE



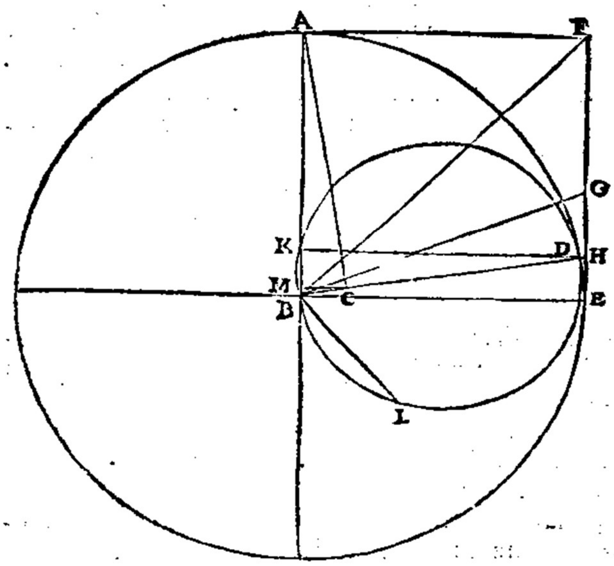
tur FBE bifariam recta linea BG. angulus igitur G BE est quarta pars vnius recti. sed DBE angulus est vnius recti pars trigesima. ergo proportio anguli GBE ad angulum DBE est ea, quam habet 15 ad 2. quarum enim partium angulus rectus est 60, earum angulus quidem GBE est 15; angulus vero D
C BE 2. Et quoniam GE ad EH maiorem proportionem habet, quàm angulus GBE ad DBE angulum; habebit GE ad EH maiorem proportionem, quàm 15 ad 2. est autem BE ęqualis EF: atque est angulus qui ad E rectus. quadratum igitur ex F Bdu-



be bisected by the straight line BG, then the angle GBE is one fourth part of a right angle, but DBE is also one thirtieth part of a right angle, therefore the ratio of the angle GBE to the angle DBE is that which 15 has to 2; if we divided a right angle into 60 equal parts then the angle GBE is made up of 15 of those parts, while the angle DBE of 2. Since GE has to EH a ratio greater than that which the angle GBE has to the angle DBE[C], therefore GE will have to EH a ratio greater than that which 15 has to 2; but since BE is also equal to EF, at the angle at E is right, therefore the square on FB is double

B duplũ eſt quadrati ex BE . vt aũt quadratũ ex FB
ad quadratũ ex BE, ita quadratũ ex FG ad quadra- D
tũ ex GE. ergo quadratũ ex FG quadrati ex GE du
plũ erit. ſed 49 minora ſunt quã dupla 25. quadratũ
igitur ex FG ad quadratum ex GE maiorem pro-
portionem habet, quàm 49 ad 25. ac propterea ipſa
FG ad GE maiorem habet proportionem , quàm 7
ad 5: & componẽdo FE ad EG maiorem , quàm 12
ad 5: hoc eſt, quàm 36 ad 15. oſtenſum autem eſt &
GE ad EH maiorem proportionem habere, quàm
15 ad 2. ergo ex æquali FE ad EH maiorem habebit
proportionem , quàm 36 ad 2 , hoc eſt quàm 18 ad
1. & ob id FE maior eſt, quàm duodeuigintuqla ip-
ſius EH. eſt autem FE æqualis EB. ergo & BE ipſius
EH maior eſt, quàm duodeuigintupla . multo igi-
tur maior erit BH , quàm duodeuigintupla ipſius E
HE. ſed vt BH ad HE, ita eſt AB ad BC ob ſimilitu- F
dinem triangulorum . ergo & AB ipſius BC maior
eſt , quàm duodeuigintupla: eſtque AB quidem di-
ſtantia , qua ſol à terra diſtat: CB vero diſtãtia qua
luna diſtat à terra: diſtantia igitur qua ſol à terra di
ſtat, diſtantiæ qua luna diſtat à terra maior eſt , quã
duodeuigintupla. Dico etiam minorem eſſe, quàm
vigintuplam. Ducatur enim per D ipſi EB paralle-
la DK, & circa DKB triangulum circulus deſcriba-
tur DKB . erit ipſius diameter DB, propterea quòd
angulus ad K rectus ſit: & aptetur BL hexagoni la-
tus . Quoniam igitur angulus DBE eſt trigeſima
pars recti , erit & B D K recti pars trigeſima . er-
go circumferentia BK ſexageſima pars eſt totius G
circuli . eſt autem & BL totius circuli pars ſexta: cir
cumferentia igitur BL decupla erit circumferentiæ
BK: atque habet circumferentia BL ad circumferen
tiam

of the square on BE. But as the square on FB is to the square on BE, so is the square on FG to the square on GE[D]; therefore the square on FG will be double of the square on GE. But 49 is less than double of 25, so that the square on FG has to the square on GE a ratio greater than that which 49 to 25 and FG itself has to GE a ratio greater than that 7 has to 5[38]; but, componendo, FE has to EG a ratio greater than that which 12 has to 5, that is, than that which 36 has to 15, then it was proved that also GE has to EH a ratio greater than that which 15 has to 2; therefore for direct proportionality, FE will have to EH a ratio greater than that which 36 has 2, that is, than that 18 has to 1, for this reason FE is greater than 18 times EH; now FE is equal to EB, therefore BE is also greater than 18 times EH, therefore BH will be much greater than 18 times[E]; but as BH is to HE, so is AB to BC because of the similarity of the triangles[F]; therefore AB is also greater than 18 times BC; but AB is the distance of the sun from the earth, as CB is the distance of the moon from the earth, therefore the distance of the sun from the earth is greater than 18 times the distance of the moon from the earth. I say that it is also less than 20 times that distance. Let DK be drawn through D parallel to EB, and about the triangle DKB let also the circle DKB be described, then DB will be its diameter because the angle at K is right; and let BL be fitted into the circle as the side of a hexagon. Then since the angle DBE is one thirtieth part of a right angle, the angle DBK is also one thirtieth part of a right angle; therefore the arc BK is one sixtieth part of the whole circle, but also BL is one six part of the whole circle therefore BL is then times the arc BK, but the arc BL has to the arc BK a ratio greater than that which the straight line BL has to the

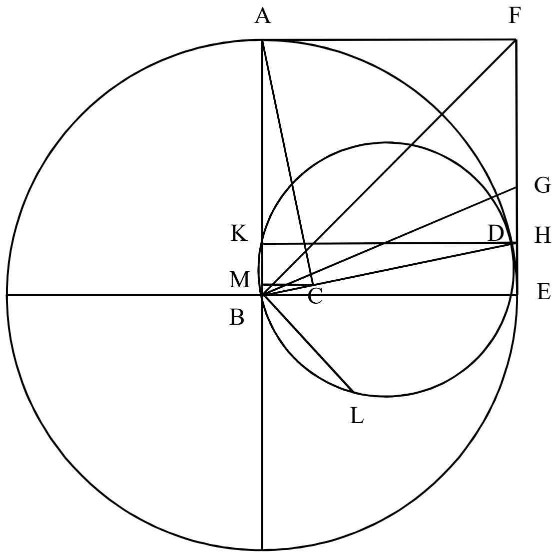


tiam BK maiorem proportionem, quàm recta linea BL ad BK rectam. ergo recta BL rectæ BK minor est, quàm decupla. est autem ipsius BL dupla B

H D. quare BD ipsius BK minor erit, quàm vigintupla. sed vt DB ad BK, ita AB ad BC. ergo & AB mi-

K nor erit, quàm vigintupla ipsius BC. estque AB quidem distantia, qua sol à terra distat; BC vero distantia, qua luna distat à terra. distantia igitur qua sol a terra distat distantię, qua luna distat à terra minor est, quàm vigintupla. ostensa autem est maior; quã duodeuigintupla. quod ostendere oportebat.

F E D.



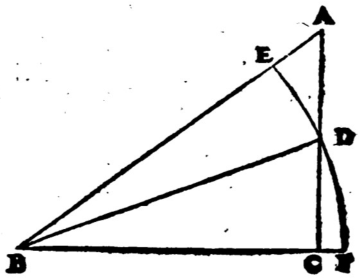
straight line BK[G], therefore the straight line BL is less than the increased tenfold BK; then BD is double of BL[H]; therefore BD will be less than 20 times BK; but, as DB is to BK , so is AB to BC[K]. Therefore AB will also less tha 20 times BC; but AB is the distance of the sun from the earth, while BC is the distance of the moon from the earth; therefore the distance of the sun from the earth is less than 20 times the distance of the moon from the earth, but it was before proved that it is greater than 18 times that distance, as it was necessary to prove.

FED. COMMANDINVS.

Ergo circumferentia ED erit trigesima pars circumferentię EDA] *Hoc in figura ita esse ponatur, namque ob loci angustiam coacti sumus circumferētiam DE multo maiorem facere, quàm sit trigesima pars circumferentiae EDA.* A

Compleatur parallelogrammum AE, & BF iungatur] *Producatur etiam BD ad rectam lineam FE in H.* B

Et quoniam GE ad EH maiorem proportionem habet, quàm angulus GBE ad DBE angulum] *Illud nos hoc lemmate demonstrabimus.* Sit triangulum orthogonium ABC rectum habens angulum ad C: & in recta linea AC sumatur quodvis punctum D, & BD iungatur. Dico rectam lineā AC ad rectam CD maiorem proportionem habere, quàm angulus ABC habeat ad DBC angulum] C



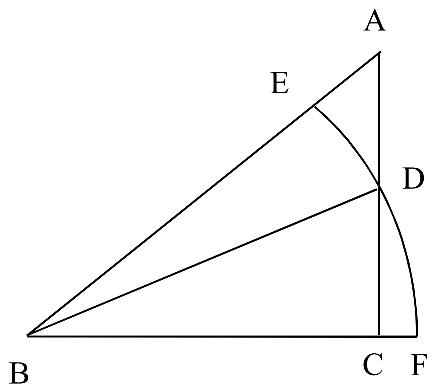
Centro enim B & interuallo BD circuli circumferentia EDF describatur, & BC producatur ad F. Itaque quoniam triangulum quidem ABD maius est sectore EBD; triangulū vero DBC minus sectore DBF: habebit triangulum ABD ad triangulum DBC maiorem proportionem, quàm sector EBD ad sectorē DBF. vt autem triangulum ABD ad triangulum

DBC

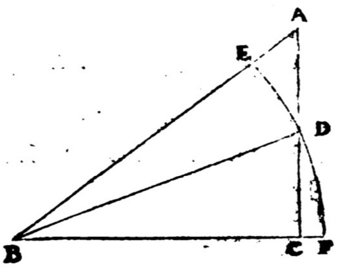
Federico Commandino

A. Then the arc ED will be one-thirtieth part of arc*: so we represent this in the picture, indeed, due to the lack of space, we are forced to represent the DE arc much larger than the 30th part of the EDA arc.*

B. Let the parallelogram AE be completed and let B&F be joined: *and let also BD produced up to H on the straight line FE.*



C. Since GE has to EH a ratio greater than that which the angle GBE has to the angle DBE: *we will prove with this lemma:* let ABC be a triangle at straight angle to C and let assume on the straight line AC any point D and let be D&B joined. I say that the straight line AC has a ratio greater to the straight line, CD, than that which the angle ABC has to the angle DBC. *Indeed let the arc EDF be drawn with center at B and radius BD and let BC be produced up to F; so, since the triangle ABD is greater than the sector EBD, then the triangle DBC is less than the sector DBF, the triangle ABD will be to the triangle DBC a ratio greater than that which the sector EDB has to the sector DBF; then as the triangle ADB is to the triangle DBC,* 1.sexti. Vlt.sextu. *DBC, ita est recta linea AD ad ipsam DC: & vt sector AB D ad sectorem DBC, ita angulus ABD ad DBC angulum. ergo recta linea A D ad ipsam DC maiorem proportionē habet, quā angulus ABD ad angulum DB C: & componendo recta linea A C ad ipsam CD, maiorem habet proportionē quā angulus ABC ad DBC angulum.*



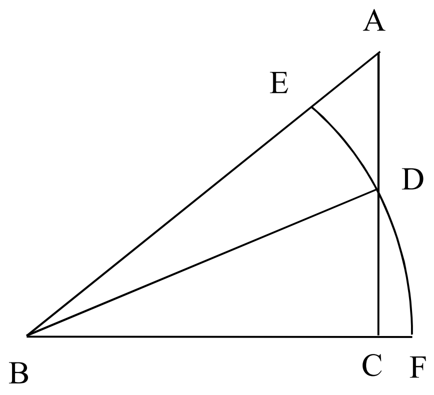
D Vt autem quadratum ex FB ad quadratum ex B E, ita quadratum ex FG ad quadratum ex GE] *Quoniam enim angulus FBE bifariam secatur recta linea BG, erit ex tertia sexti elementorum vt FB ad BE, ita FG ad G E: quare ex 22 eiusdem, vt quadratum ex FB ad quadratum ex BE, ita quadratum ex FG ad quadratum ex GE.*

E Multo igitur maior erit BH, quàm duodeuigintupla ipsius HE] *Nam BH, quę maiori angulo, nempe recto subtenditur, maior est, quàm ipsa BE.*

F Sed ut BH ad HE ita est AB ad BC, ob triangulorum similitudinem]. *Ducatur à puncto C, videlicet ab angulo recto trianguli ABC ad basim perpendicularis CM;* 8 sexti. *fient triangula BCM, ACM similia toti, & inter se se. quare* 29. primi. *angulus BCM, hoc est angulus HBE est aequalis angulo B AC. atque est ACB rectus aequalis recto BEH. reliquus igi-* 4.sexti *tur ABC reliquo BHE est aequalis, & triangulum triangulo simile. ergo vt BH ad HE, ita AB ad BC.*

G Atque habet circumferentia BL ad circumferen-

tiam



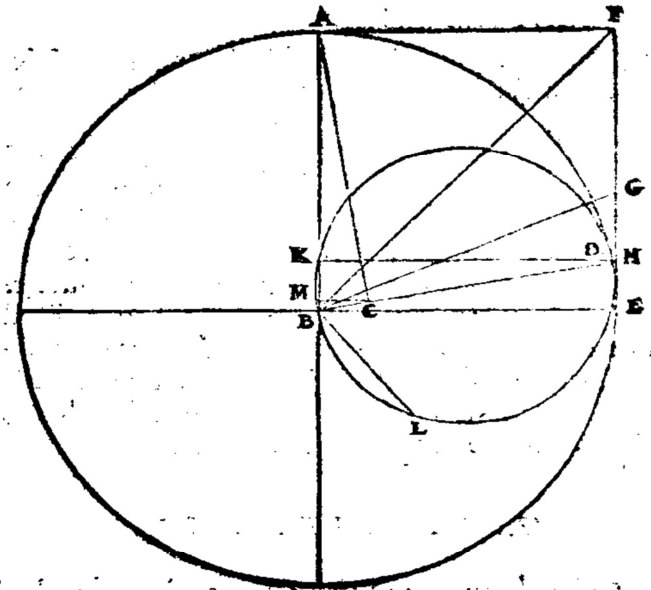
so is the straight line AD to the line DC; as theil sector ABD is to the sector DBC, so the angle ABD is to the angle DBC; then the straight line AD has to DC a ratio greater than that which the angle ABD has to the angle DBC; and, componendo, the straight line AC has to CD a ratio greater than that which the angle ABC has to the angle DBC.[39]

D. But as the square on FB is to the square on BE, so is the square on FG to the square on GE: *in fact since the angle FBE is cut in two equal parts by the straight line BG, from third proposition of sixth book of Elements, as FB is to BE so FG is to GE; wherefore from 22° of this book, as the the square on FB is to the square on BE, so the square on FG is to the square on GE.*

E. Therefore BH will be much greater than 18 times HE: *in fact BH, wich extend below the greater angle, which is right, is greater then BE.*

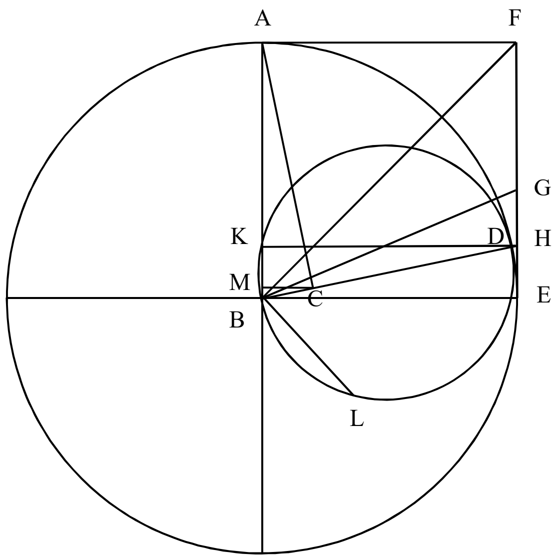
F. But as BH is to HE, so is AB to BC because of the similarity of the triangles: *let the perpendicular CM be drawn from point C to the base, that is from the right angle of the triangle AB; the triangles BCM and ACM will be completely similar between them, therefore the angle BCM, that is the angle HBE is equal to the angle BAC and the right angle ACB is equal to BEH also right, then the remaining ABC is equal to the remaining BHE, therefore the triangles are similar; then as BH is to HE, so AB is toBC.*

G. but the arc BL has to the arc BK a ratio greater than that which the straight line BL has to the tiam BK maiorem proportionem, quàm recta linea BL ad BK rectam.] *Ex demonstratis à Ptolemęo in principio magnę constructionis.*



Est autem ipsius BL dupla BD] *Ex corollario quintę decimę quarti libri elementorum.* H

Sed ut DB ad BK, ita AB ad BC] *Ob triangulorum DBK, ABC similitudinem. Rursus enim angulus MCB, hoc est BDK est aequalis angulo BAC, rectusq; DKB recto AC B, & reliquus reliquo aequalis.* K straight line BK: *from the demonstrations of Ptolemy at the beginning of Liber Magnae Costructionis.*



H. Then BD is double of BLI: *from the corollary of the fifteenth proposition of the fourth book of Elements.*

K. But, as DB is to BK, so is AB to: *by similarity of the triangles BDK and ABC; in fact again the angle MCB, that is BDK is equal to the angle BAC and the right angle BKD to the right angle ACB and also the remaining will be equal to remaining.*

NOTE

[1] *Preclarissimus liber elementorum Euclidis perspicacissimi in artem Geometrie incipit quá foelicissime.Venetijs Erhardus Ratdolt. 1482*.

[2] *Euclide megarense philosopho: solo introduttore delle scientie mathematice diligentemente rassettato, et alla integrità ridotto per il degno professore di tal scientie Nicolo Tartalea, brisciano, secondo le due tradottioni e per comune commodo & utilita di latino in volgar tradotto Stampato in Vinegia MDXLIII.*

[3] *Archimedis Syracusani Philosophi ac geometrae excellentissimi Opera, quae quidem, omnia, multis iam seculis desiderata, atque à quam paucisimis hactenus visa, nunque primum et graece et latinae in lucem edita.*

[4] s. Umberto Bottazzini. *Antichi paradigmi e nuovi metodi geometrici.* In: *Storia della scienza moderna e contemporanea.* Vol. primo. *Dalla rivoluzione scientifica all'età dei lumi.* Tomo primo. TEA 2000.

[5] s. Guido Castelnuovo: *Le origini del calcolo infinitesimale nell'era moderna.* Feltrinelli 1962.
Enrico Ruffini: *Il "Metodo" di Archimede e le origini del calcolo infinitesimale nell'antichità*. Feltrinelli 1961.

[6] "We indicate with the term of center of gravity, that particular point placed within each body, for which if one imagines with the mind that the grave is suspended, it remains stationary while being moved and also retains the position it had at the beginning: neither rotates during movement"

[7] "Now I am persuaded that if this work has pleased Your Excellency in the Latin garment, you will not mind in this our vulgar language" At the beginning of the book there is the following warning: "Admonendus es mihi, candide lector, auctorem hunc, quem tibi exibemus, Euclide usum in arabicam linguam converso, quem postea Campanus latinum fecit. Hoc dictum volui, ne in perquirendis propositionibus, quos ipse citat, quandoque te frustra excruciares.Vale."
"I must warn you, dear reader, that the author, whom we now present to you, made use of the Euclid translated into the Arabic language, then made Latin by the Campanus. And so I wanted to tell you in order that in trying propositions cited by him, no worries you sometimes vain. It's healthy."

[8] The Sand Reckoner.

[9] For a more in-depth look at the Greek manuscripts handed down to us by Aristarchus, see the work of Thomas Heath, John Wallis and Fortia d'Urban. (note 11,13,22)

[10] It is an angular measure obtained by determining the angle between the earth-moon and earth-sun segments when the angle moon-earth and moon-sun is right.

[11] The recalculation of the earth-sun distance using the Aristarchus method, but using the lunar elongation of 89 ° 51 ', is shown in note 40.

[12] Thomas L.Heath: *Aristarchus of Samos, the ancient Copernicus; a history of Greek astronomy to Aristarchus together with Aristarchus's treatise on the sizes and distances of the sun and moon a new greek text with translation and notes.* Oxford at the Clarendon Press, 1913.

[13] s. Lucio Russo*: La rivoluzione dimenticata* § 2.8. Feltrinelli, 2008

[14] The passage is transalated by: *Archimedis Syracusani Arenarius et Dimensio Circuli. Eutocii Ascalonitae in hanc Commentarius Cum versione & notis Joh. Wallis,SS.TH.D. Geometriae Professoris Saviliani Oxonii E Theatro Sheldoniano 1676* (Latin translation of the Greek text opposite). The passage presents some interpretative difficulties; for further discussion see Heath Thomas L. *Aristarchus of Samos, the ancient Copernicus* Oxford Clarendon press, 1913 p. 301 and f.

[15] To deepen the subject s.n. 12 and Pierre Duhem: *Sauver les apparences* Paris Vrin 2003

[16] First Kepler's law: "sequenti capite, ubi simul etiam demonstrabitur, nullam Planetæ relinqui figuram Orbitæ, praeterquam perfecte ellipticam; conspirantibus rationibus. a principiis Physicis, derivatis, cum experientia obserrvationum et hypotheseos vicariae hoc capite allegata*"*.
Second Kepler's law: a radius vector joining any planet centre to the centre of the sun sweeps out equal areas in equal lengths of time.

[17] *Regula I: causas rerum naturalium non plures admitti debere, quam quae et verae sint et earum phaenomenis explicandis sufficiant. Dicunt utique philosophi: Natura nihil agit frustra, & frustra sit per plura quod fieri potest per pauciora. Natura enim simplex est & rerum causis superfluis non luxuriat.*

[18] The greek notation is taken by Heath (s. n.11), by Fortia D'Urban (s.n. 22) the same nambers are shown as follows: ρκε`θ,ψιβ´=1259712, `ζθ,φζ´=79507. Commandino does not say from which manuscript sources he transalated the text.

[19] De Arenae Numero or Arenarius.

[20] The sphere in which the moon moves.

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[21] Fortia D'Urban thus translates: "Lorsque la lune nous parait *dikhotome* (coupée en deux portions égales), elle offre à nos regards son grand cercle, qui détermine la partie éclairée et la partie obscure de cet aster". However it seems clear to us that for circulum maximum we must understand the flat figure and not its perimeter, so it is the plane of the maximum circle that passes through our point of view. Commandino in fact distinguishes circulus from circumferentia.
Fortia D'Urban: *Traité d'Aristarque de Samos, sur les grandeurs et les distances du soleil et de la lune et fragment de Héron de Bisance sur le mesures.* Paris, Firmin Didot. 1823.

[22] Earth's shadow.

[23] s.n. 23

[24] 2°

[25] s.n. 23

[26] On the book *Pappi Alexandrini Mathematicae Collectiones a Federico Commandino in Latinum Conversae et Commentariis Illustratae. Bononiae ex Typographia H.H.de Ducijs MDCLX* it is given 0.40.40

[27] 0°17'33"

[28] 0°5'30"

[29] It is correctly given in the book of the note 27: *diametri autem terrae quintupla, et adhuc dimidio maior.* "diameter" this is an obvious mistake.

[30] No distinction is made in the text between the straight line and the segment of it.

[31] On the extension of AB.

[32] If A:B=C:D then (A+B):B=(C+D):D

[33] *De iis quae vehuntur in aqua libri duo*: Treatise on floating bodies galleggianti.

[34] On text *B* (obvious mistake)

[35] On text *DO* (obvious mistake)

[36] You mean: below the sphere of the sun or, if you prefer, it has an orbit contained within that of the sun.

[37] If we try to repeat the reasoning of Aristarchus by hypothesizing that the lunar elongation in quadrature is not 87 ° but 89 ° 51 '(ie 5391'), the distance of the sun from the earth falls between 360 and 400 times the distance of the moon from the earth. This result is close to the currently accepted average one.

For instance:

4. Hypothesis

When the moon appears us halved, its distance from the sun is then less than a quadrant by six hundredth of a quadrant (or 9'), i.e. 5391 parts, these in fact differ from nine parts, which are the seventeenth part of 5400, by 5400 parts of a quadrant.

PROPOSITION VII

The distance that separates the sun from the earth is greater than three hundred and sixty times, but less than four hundred times, the distance of the moon from the earth.

Let A be the centre of the sun, whilst B that of the earth B, let A& B joined and produced; let C be the centre of the moon when halved, let a plane be carried through AB and C which cut the sphere and let be the great circle ADE thath on which the centre of the sun moves, and let A&C and C&B be joined and let BC be produced to D. The angle ACB will be certainly right, because the point C is the centre of the moon when halved; let BE be drawn from B at right angles to BA itself, then the arc ED will be one-thirtieth part of arc EDA; indeed, by hypothesis, when the moon appears to us halved, its distance from the sun is less than a quadrant by one-thirtieth of a quadrant, therefore the angle BC is also one-thirtieth part of a right angle. Let the parallelogram AE be completed and let B&F be joined, the angle FBE will be half a right angle. Let the angle FBE be bisected by the straight-line BG, then the angle GBE is one fourth part of a right angle, but DBE is also one six hundredth part of a right angle, therefore the ratio of the angle GBE to the angle DBE is than which 15 has to 1/10; if we divided a right angle into 60 equal parts then the angle GBE is made up of 15 of those parts, while the angle DBE of 1/10. Since GE has to EH a ratio greater than that which the angle GBE has to the angle DBE, therefore GE will have to EH a ratio greater than that which 15 has to 1/10; but since BE is also equal to EF, at the angle at E is right; therefore the square on FB is double of the square on BE. But as the square on FB is to the square on BE, so is the square on FG to the square on GE; therefore the square on FG will be double of the square on GE. But 49 is less than double of 25, so that the square on FG has to the square on GE a ratio greater than that which 49 to 25 and FG itself has to GE a ratio greater than that 7 has to 5; but componendo FE has to EG a ratio greater than that which 12 has to 5, that is, than that which 36 has to 15, then it was proved that also GE has to EH a ratio greater than that which 15 has to 1/10; therefore for direct proportionality, FE will have to EH a ratio greater than that which 36 has 1/10, that is, than that 360 has to 1, for this reason FE is greater than 360 times EH; now FE is equal to EB, therefore BE is also greater than 360 times EH, therefore BH will be much greater than 360 times; but as BH is to HE, so is AB to BC because of the similarity of the triangles; therefore AB is also greater than 360 times BC; but AB is the distance of the sun from the earth, as CB is the distance of the moon from the earth, therefore the distance of the sun from the earth is greater than 360 times the distance of the moon from the eart. I say that it is also less than 400 times that distance. Let DK be drawn through D parallel to EB, and about the triangle DKB let also the circle DKB be described, then DB will be its diameter because the angle at K is right; and let BL be fitted into the circle as the side of a hexagon. Then since the angle DBE is one six hundredth part of a right angle, the angle DBK is also one six hundredth part of a right angle; therefore the arc BK is one sixtieth part of the whole circle, but also BL is one six part of the whole circle therefore BL is two hundred times the arc BK, but the arc BL has to the arc BK a ratio greater than that which the straight line BL has to the straight line BK, therefore the straight line BL is less than the increased two hundredfold BK; then BD is double of BL; therefore BD will be less than 400 times BK; but, as DB is to BK , so is AB to BC. Therefore AB will also less tha 400 times BC; but AB is the distance of the sun from the earth, while BC is the distance of the moon from the earth; therefore the distance of the sun from the earth is less than 400 times the distance of the moon from the earth, but it was before proved that it is greater than 360 times that distance, as it was necessary to prove.

38 The Pythagoras theorem states that the square of the hypotenuse is equal to the sum of the squares of the catheti. In the case of triangle FEB the catheti are equals, consequently, if the value 5 is assigned to the catheti, the square on the hypotenuse will be 50, but the number 49, which is earlier, has as its square root 7. This allows an approximate expression of the square root of 2 (50 =25x2, $\sqrt{50} = \sqrt{25x2} = 5\sqrt{2}$, but $\sqrt{50} \sim 7$ therefore $7 \sim 5\sqrt{2}$, then $\frac{7}{5} \sim \sqrt{2}$).

39 As demonstrated by Commandino can be expressed with the known proposition: ($ABC > DBC < 90°$)